

Advanced Topics

Harvard-MIT Math Tournament

February 27, 1999

1. One of the receipts for a math tournament showed that 72 identical trophies were purchased for \$-99.9-, where the first and last digits were illegible. How much did each trophy cost?
2. Stacy has d dollars. She enters a mall with 10 shops and a lottery stall. First she goes to the lottery and her money is doubled, then she goes into the first shop and spends 1024 dollars. After that she alternates playing the lottery and getting her money doubled (Stacy always wins) then going into a new shop and spending \$1024. When she comes out of the last shop she has no money left. What is the minimum possible value of d ?
3. An unfair coin has the property that when flipped four times, it has the same nonzero probability of turning up 2 heads and 2 tails (in any order) as 3 heads and 1 tail (in any order). What is the probability of getting a head in any one flip?
4. You are given 16 pieces of paper numbered 16, 15, ..., 2, 1 in that order. You want to put them in the order 1, 2, ..., 15, 16 switching only two adjacent pieces of paper at a time. What is the minimum number of switches necessary?
5. For any finite set S , let $f(S)$ be the sum of the elements of S (if S is empty then $f(S) = 0$). Find the sum over all subsets E of S of $\frac{f(E)}{f(S)}$ for $S = \{1, 2, \dots, 1999\}$.
6. Matt has somewhere between 1000 and 2000 pieces of paper he's trying to divide into piles of the same size (but not all in one pile or piles of one sheet each). He tries 2, 3, 4, 5, 6, 7, and 8 piles but ends up with one sheet left over each time. How many piles does he need?
7. Find an ordered pair (a, b) of real numbers for which $x^2 + ax + b$ has a non-real root whose cube is 343.
8. Let C be a circle with two diameters intersecting at an angle of 30 degrees. A circle S is tangent to both diameters and to C , and has radius 1. Find the largest possible radius of C .
9. As part of his effort to take over the world, Edward starts producing his own currency. As part of an effort to stop Edward, Alex works in the mint and produces 1 counterfeit coin for every 99 real ones. Alex isn't very good at this, so none of the counterfeit coins are the right weight. Since the mint is not perfect, each coin is weighed before leaving. If the coin is not the right weight, then it is sent to a lab for testing. The scale is accurate 95% of the time, 5% of all the coins minted are sent to the lab, and the lab's test is accurate 90% of the time. If the lab says a coin is counterfeit, what is the probability that it really is?
10. Find the minimum possible value of the largest of xy , $1 - x - y + xy$, and $x + y - 2xy$ if $0 \leq x \leq y \leq 1$.