Algebra
Rice Mathematics Tournament 2000

1. How many integers x satisfy $|x| + 5 < 7$ and $|x - 3| > 2$?

2. Evaluate $2000^3 - 1999 \cdot 2000^2 - 1999^2 \cdot 2000 + 1999^3$.

3. Five students take a test on which any integer score from 0 to 100 inclusive is possible. What is the largest possible difference between the median and the mean of the scores?

4. What is the fewest number of multiplications required to reach x^{2000} from x, using only previously generated powers of x? For example, $x \to x^2 \to x^4 \to x^8 \to x^{16} \to x^{32} \to x^{64} \to x^{128} \to x^{256} \to x^{512} \to x^{1024} \to x^{1536} \to x^{1792} \to x^{1920} \to x^{1984} \to x^{2000}$ uses 15 multiplications.

5. A jacket was originally priced \$100. The price was reduced by 10% three times and increased by 10% four times in some order. To the nearest cent, what was the final price?

6. Barbara, Edward, Abhinav, and Alex took turns writing this test. Working alone, they could finish it in 10, 9, 11, and 12 days, respectively. If only one person works on the test per day, and nobody works on it unless everyone else has spent at least as many days working on it, how many days (an integer) did it take to write this test?

7. A number n is called multiplicatively perfect if the product of all the positive divisors of n is n^2. Determine the number of positive multiplicatively perfect numbers less than 100.

8. A man has three daughters. The product of their ages is 168, and he remembers that the sum of their ages is the number of trees in his yard. He counts the trees but cannot determine any of their ages. What are all possible ages of his oldest daughter?

9. $\frac{a}{c} = \frac{b}{d} = \frac{3}{4}$, $\sqrt{a^2 + c^2} - \sqrt{b^2 + d^2} = 15$. Find $ac + bd - ad - bc$.

10. Find the smallest positive integer a such that $x^4 + a^2$ is not prime for any integer x.