Calculus
Rice Mathematics Tournament 2000

1. Find the slope of the tangent at the point of inflection of $y = x^3 - 3x^2 + 6x + 2000$.
2. Karen is attempting to climb a rope that is not securely fastened. If she pulls herself up x feet at once, then the rope slips x^3 feet down. How many feet at a time must she pull herself up to climb as efficiently as possible?
3. A rectangle of length $\frac{1}{4}\pi$ and height 4 is bisected by the x-axis and is in the first and fourth quadrants. The graph of $y = \sin(x) + C$ divides the area of the square in half. What is C?
4. For what value of x $(0 < x < \frac{\pi}{2})$ does $\tan x + \cot x$ achieve its minimum?
5. For $-1 < x < 1$, let $f(x) = \sum_{i=1}^{\infty} \frac{x^i}{i}$. Find a closed form expression (a closed form expression is one not involving summation) for f.
6. A hallway of width 6 feet meets a hallway of width $6\sqrt{5}$ feet at right angles. Find the length of the longest pipe that can be carried horizontally around this corner.
7. An **envelope** of a set of lines is a curve tangent to all of them. What is the envelope of the family of lines $y = \frac{2}{x_0} + x(1 - \frac{1}{x_0^2})$, with x_0 ranging over the positive real numbers? *Hint*: slope at a point P on a curve is $\frac{dy}{dx}|_P$.
8. Find $\int_0^{\frac{\pi}{2}} \ln \sin \theta d\theta$.
9. Let $f(x) = \sqrt{x + \sqrt{0 + \sqrt{x + \sqrt{0 + \sqrt{x + \ldots}}}}}$. If $f(a) = 4$, then find $f'(a)$.
10. A mirror is constructed in the shape of y equals $\pm\sqrt{x}$ for $0 \le x \le 1$, and ± 1 for $1 < x < 9$. A ray of light enters at (10,1) with slope 1. How many times does it bounce before leaving?