

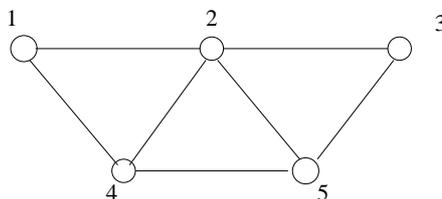
General

Rice Mathematics Tournament 2000

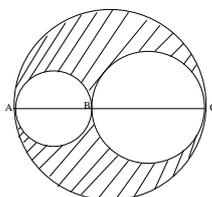
1. If $a = 2b + c$, $b = 2c + d$, $2c = d + a - 1$, $d = a - c$, what is b ?
2. The temperatures $f^\circ\text{F}$ and $c^\circ\text{C}$ are equal when $f = \frac{9}{5}c + 32$. What temperature is the same in both $^\circ\text{F}$ and $^\circ\text{C}$?
3. A twelve foot tree casts a five foot shadow. How long is Henry's shadow (at the same time of day) if he is five and a half feet tall?
4. Tickets for the football game are \$10 for students and \$15 for non-students. If 3000 fans attend and pay \$36250, how many students went?
5. Find the interior angle between two sides of a regular octagon (degrees).
6. Three cards, only one of which is an ace, are placed face down on a table. You select one, but do not look at it. The dealer turns over one of the other cards, which is not the ace (if neither are, he picks one of them randomly to turn over). You get a chance to change your choice and pick either of the remaining two face-down cards. If you selected the cards so as to maximize the chance of finding the ace on the second try, what is the probability that you selected it on the
 - (a) first try?
 - (b) second try?
7. Find $\lceil \sqrt{19992000} \rceil$ where $[x]$ is the greatest integer less than or equal to x .
8. Bobo the clown was juggling his spherical cows again when he realized that when he drops a cow is related to how many cows he started off juggling. If he juggles 1, he drops it after 64 seconds. When juggling 2, he drops one after 55 seconds, and the other 55 seconds later. In fact, he was able to create the following table:

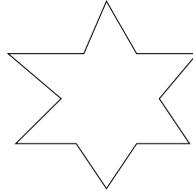
cows started juggling	1	2	3	4	5	6	7	8	9	10	11
seconds he drops after	64	55	47	40	33	27	22	18	14	13	12
cows started juggling	12	13	14	15	16	17	18	19	20	21	22
seconds he drops after	11	10	9	8	7	6	5	4	3	2	1
- He can only juggle up to 22 cows. To juggle the cows the longest, what number of cows should he start off juggling? How long (in minutes) can he juggle for?
9. Edward's formula for the stock market predicts correctly that the price of HMMT is directly proportional to a secret quantity x and inversely proportional to y , the number of hours he slept the night before. If the price of HMMT is \$12 when $x = 8$ and $y = 4$, how many dollars does it cost when $x = 4$ and $y = 8$?
10. Bob has a 12 foot by 20 foot garden. He wants to put fencing around it to keep out the neighbor's dog. Normal fenceposts cost \$2 each while strong ones cost \$3 each. If he needs one fencepost for every 2 feet and has \$70 to spend on the fenceposts, what is the largest number of strong fenceposts he can buy?
11. If $a@b = \frac{a+b}{a-b}$, find n such that $3@n = 3$.

12. In 2020, the United States admits North Mathematica as the 51st state. It consists of 5 islands joined by bridges as shown. Is it possible to cross all the bridges without doubling over? If so, what is the difference (positive) between the number of the start island and the number of the end island?



13. How many permutations of 123456 have exactly one number in the correct place?
14. The author of this question was born on April 24, 1977. What day of the week was that?
15. Which is greater: $(3^5)^{(5^3)}$ or $(5^3)^{(3^5)}$?
16. Joe bikes x miles East at 20 mph to his friend's house. He then turns South and bikes x miles at 20 mph to the store. Then, Joe turns East again and goes to his grandma's house at 14 mph. On this last leg, he has to carry flour he bought for her at the store. Her house is 2 more miles from the store than Joe's friend's house is from the store. Joe spends a total of 1 hour on the bike to get to his grandma's house. If Joe then rides straight home in his grandma's helicopter at 78 mph, how many minutes does it take Joe to get home from his grandma's house?
17. In how many distinct ways can the letters of STANTON be arranged?
18. You use a lock with four dials, each of which is set to a number between 0 and 9 (inclusive). You can never remember your code, so normally you just leave the lock with each dial one higher than the correct value. Unfortunately, last night someone changed all the values to 5. All you remember about your code is that none of the digits are prime, 0, or 1, and that the average value of the digits is 5. How many combinations will you have to try?
19. Eleven pirates find a treasure chest. When they split up the coins in it, they find that there are 5 coins left. They throw one pirate overboard and split the coins again, only to find that there are 3 coins left over. So, they throw another pirate over and try again. This time, the coins split evenly. What is the least number of coins there could have been?
20. Given: AC has length 5, semicircle AB has radius 1, semicircle BC has diameter 3. What percent of the the big circle is shaded?





21. Find the area of the six-pointed star if all edges are of length s , all acute angles are 60° and all obtuse angles are 240° .
22. An equilateral triangle with sides of length 4 has an isosceles triangle with the same base and half the height cut out of it. Find the remaining area.
23. What are the last two digits of 7^{7^7} ?
24. Peter is randomly filling boxes with candy. If he has 10 pieces of candy and 5 boxes in a row labeled A, B, C, D, and E, how many ways can he distribute the candy so that no two adjacent boxes are empty?
25. How many points does one have to place on a unit square to guarantee that two of them are strictly less than $1/2$ unit apart?
26. Janet is trying to find Tim in a Cartesian forest. Janet is $5\sqrt{2}$ miles from $(0,0)$, $\sqrt{41}$ miles from $(1,0)$, and $\sqrt{61}$ miles from $(0,1)$. Tim is $\sqrt{65}$ miles from $(0,0)$, $2\sqrt{13}$ miles from $(1,0)$, and $\sqrt{58}$ miles from $(0,1)$. How many miles apart are Janet and Tim?