# Team Test
## Rice Mathematics Tournament 2000

1. You are given a number, and round it to the nearest thousandth, round this result to nearest hundredth, and round this result to the nearest tenth. If the final result is .7, what is the smallest number you could have been given? As is customary, 5's are always rounded up. Give the answer as a decimal.

2. The price of a gold ring in a certain universe is proportional to the square of its purity and the cube of its diameter. The purity is inversely proportional to the square of the depth of the gold mine and directly proportional to the square of the price, while the diameter is determined so that it is proportional to the cube root of the price and also directly proportional to the depth of the mine. How does the price vary solely in terms of the depth of the gold mine?

3. Find the sum of all integers from 1 to 1000 inclusive which contain at least one 7 in their digits, i.e. find $7 + 17 + ... + 979 + 987 + 997$.

4. All arrangements of letters VNNWHTAAIE are listed in lexicographic (dictionary) order. If AAEHINNTVW is the first entry, what entry number is VANNAWHITE?

5. Given $\cos(\alpha + \beta) + \sin(\alpha - \beta) = 0$, $\tan\beta = \frac{1}{2000}$, find $\tan\alpha$.

6. If $\alpha$ is a root of $x^3 - x - 1 = 0$, compute the value of $\alpha^{10} + 2\alpha^8 - \alpha^7 - 3\alpha^6 - 3\alpha^5 + 4\alpha^4 + 2\alpha^3 - 4\alpha^4 - 6\alpha - 17$.

7. 8712 is an integral multiple of its reversal, 2178, as 8712=4*2178. Find another 4-digit number which is a non-trivial integral multiple of its reversal.

8. A woman has \$1.58 in pennies, nickels, dimes, quarters, half-dollars and silver dollars. If she has a different number of coins of each denomination, how many coins does she have?

9. Find all positive primes of the form $4x^4 + 1$, for $x$ an integer.

10. How many times per day do at least two of the three hands on a clock coincide?

11. Find all polynomials $f(x)$ with integer coefficients such that the coefficients of both $f(x)$ and $[f(x)]^3$ lie in the set $\{0, 1, -1\}$

12. At a dance, Abhinav starts from point $(a, 0)$ and moves along the negative x direction with speed $v_a$, while Pei-Hsin starts from $(0, b)$ and glides in the negative y-direction with speed $v_b$. What is the distance of closest approach between the two?

13. Let $P_1, P_2, \ldots, P_n$ be a convex $n$-gon. If all lines $P_iP_j$ are joined, what is the maximum possible number of intersections in terms of $n$ obtained from strictly inside the polygon?

14. Define a sequence $< x_n >$ of real numbers by specifying an initial $x_0$ and by the recurrence $x_{n+1} = \frac{1+x_n}{1-x_n}$. Find $x_n$ as a function of $x_0$ and $n$, in closed form. There may be multiple cases.

15. $\lim_{n\to\infty} nr\sqrt[2]{1 - \cos\frac{2\pi}{n}} = ?$