Geometry Test

Harvard-MIT Math Tournament

March 3, 2001

1. A circle of radius 3 crosses the center of a square of side length 2. Find the positive difference between the areas of the nonoverlapping portions of the figures.

2. Call three sides of an opaque cube adjacent if someone can see them all at once. Draw a plane through the centers of each triple of adjacent sides of a cube with edge length 1. Find the volume of the closed figure bounded by the resulting planes.

3. Square $ABCD$ is drawn. Isosceles Triangle CDE is drawn with E a right angle. Square $DEFG$ is drawn. Isosceles triangle FGH is drawn with H a right angle. This process is repeated infinitely so that no two figures overlap each other. If square $ABCD$ has area 1, compute the area of the entire figure.

4. A circle has two parallel chords of length x that are x units apart. If the part of the circle included between the chords has area $2 + \pi$, find x.

5. Find the volume of the tetrahedron with vertices $(5, 8, 10)$, $(10, 10, 17)$, $(4, 45, 46)$, $(2, 5, 4)$.

6. A point on a circle inscribed in a square is 1 and 2 units from the two closest sides of the square. Find the area of the square.

7. Equilateral triangle ABC with side length 1 is drawn. A square is drawn such that its vertex at A is opposite to its vertex at the midpoint of BC. Find the area enclosed within the intersection of the insides of the triangle and square. Hint: $\sin 75 = \frac{\sqrt{2}(\sqrt{3}+1)}{4}$.

8. Point D is drawn on side BC of equilateral triangle ABC, and AD is extended past D to E such that angles EAC and EBC are equal. If $BE = 5$ and $CE = 12$, determine the length of AE.

9. Parallelogram $AECF$ is inscribed in square $ABCD$. It is reflected across diagonal AC to form another parallelogram $AE'CF'$. The region common to both parallelograms has area m and perimeter n. Compute the value of $\frac{m}{n^2}$ if $AF : AD = 1 : 4$.

10. A is the center of a semicircle, with radius AD lying on the base. B lies on the base between A and D, and E is on the circular portion of the semicircle such that EBA is a right angle. Extend EA through A to C, and put F on line CD such that EBF is a line. Now $EA = 1$, $AC = \sqrt{2}$, $BF = \frac{2-\sqrt{2}}{4}$, $CF = \frac{2\sqrt{5}+\sqrt{10}}{4}$, and $DF = \frac{2\sqrt{5}-\sqrt{10}}{4}$. Find DE.