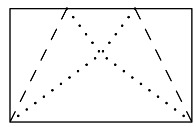
12^{th} Annual Harvard-MIT Mathematics Tournament

Saturday 21 February 2009

Individual Round: Geometry Test

1. [**3**] A rectangular piece of paper with side lengths 5 by 8 is folded along the dashed lines shown below, so that the folded flaps just touch at the corners as shown by the dotted lines. Find the area of the resulting trapezoid.



2. [**3**] The corner of a unit cube is chopped off such that the cut runs through the three vertices adjacent to the vertex of the chosen corner. What is the height of the cube when the freshly-cut face is placed on a table?

3. [**4**] Let T be a right triangle with sides having lengths 3, 4, and 5. A point P is called *awesome* if P is the center of a parallelogram whose vertices all lie on the boundary of T. What is the area of the set of awesome points?

4. [**4**] A *kite* is a quadrilateral whose diagonals are perpendicular. Let kite $ABCD$ be such that $\angle B = \angle D = 90°$. Let M and N be the points of tangency of the incircle of $ABCD$ to AB and BC respectively. Let ω be the circle centered at C and tangent to AB and AD. Construct another kite $AB'C'D'$ that is similar to $ABCD$ and whose incircle is ω. Let N' be the point of tangency of $B'C'$ to ω. If $MN' \parallel AC$, then what is the ratio of $AB : BC$?

5. [**4**] Circle B has radius $6\sqrt{7}$. Circle A, centered at point C, has radius $\sqrt{7}$ and is contained in B. Let L be the locus of centers C such that there exists a point D on the boundary of B with the following property: if the tangents from D to circle A intersect circle B again at X and Y, then XY is also tangent to A. Find the area contained by the boundary of L.

6. [**5**] Let ABC be a triangle in the coordinate plane with vertices on lattice points and with $AB = 1$. Suppose the perimeter of ABC is less than 17. Find the largest possible value of $1/r$, where r is the inradius of ABC.

7. [**5**] In triangle ABC, D is the midpoint of BC, E is the foot of the perpendicular from A to BC, and F is the foot of the perpendicular from D to AC. Given that $BE = 5$, $EC = 9$, and the area of triangle ABC is 84, compute $|EF|$.

8. [**7**] Triangle ABC has side lengths $AB = 231, BC = 160$, and $AC = 281$. Point D is constructed on the opposite side of line AC as point B such that $AD = 178$ and $CD = 153$. Compute the distance from B to the midpoint of segment AD.

9. [**7**] Let ABC be a triangle with $AB = 16$ and $AC = 5$. Suppose the bisectors of angles $\angle ABC$ and $\angle BCA$ meet at point P in the triangle's interior. Given that $AP = 4$, compute BC.

10. [**8**] Points A and B lie on circle ω. Point P lies on the extension of segment AB past B. Line ℓ passes through P and is tangent to ω. The tangents to ω at points A and B intersect ℓ at points D and C respectively. Given that $AB = 7, BC = 2$, and $AD = 3$, compute BP.