THE HARVARD-MIT MATHEMATICS TOURNAMENT

GEOMETRY TEST

This test consists of 10 short-answer problems to be solved individually in 50 minutes. Problems will be weighted with point values after the contest based on how many competitors solve each problem. There is no penalty for guessing.

No translators, books, notes, slide rules, calculators, abaci, or other computational aids are permitted. Similarly, graph paper, rulers, protractors, compasses, and other drawing aids are not permitted.

Answers should be simplified as much as is reasonably possible and must be exact unless otherwise specified. Rational numbers should be written in lowest terms, although denominators of irrationals need not be rationalized. An nth root should be simplified so that the radicand is not divisible by the nth power of any prime.

Correct mathematical notation must be used. No partial credit will be given unless otherwise specified.

If you believe the test contains an error, please submit your protest in writing to Lobby 10 during lunchtime.

Enjoy!

HMMT 2013

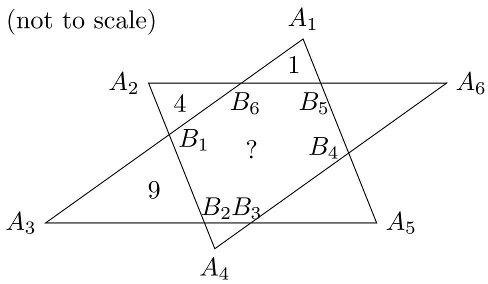
Saturday 16 February 2013

Geometry Test

1. Jarris the triangle is playing in the (x, y) plane. Let his maximum y coordinate be k. Given that he has side lengths 6, 8, and 10 and that no part of him is below the x-axis, find the minimum possible value of k.

2. Let $ABCD$ be an isosceles trapezoid such that $AD = BC$, $AB = 3$, and $CD = 8$. Let E be a point in the plane such that $BC = EC$ and $AE \perp EC$. Compute AE.

3. Let $A_1A_2A_3A_4A_5A_6$ be a convex hexagon such that $A_iA_{i+2} \parallel A_{i+3}A_{i+5}$ for $i = 1, 2, 3$ (we take $A_{i+6} = A_i$ for each i). Segment A_iA_{i+2} intersects segment $A_{i+1}A_{i+3}$ at B_i, for $1 \le i \le 6$, as shown. Furthermore, suppose that $\triangle A_1A_3A_5 \cong \triangle A_4A_6A_2$. Given that $[A_1B_5B_6] = 1$, $[A_2B_6B_1] = 4$, and $[A_3B_1B_2] = 9$ (by $[XYZ]$ we mean the area of $\triangle XYZ$), determine the area of hexagon $B_1B_2B_3B_4B_5B_6$.



4. Let ω_1 and ω_2 be circles with centers O_1 and O_2, respectively, and radii r_1 and r_2, respectively. Suppose that O_2 is on ω_1. Let A be one of the intersections of ω_1 and ω_2, and B be one of the two intersections of line O_1O_2 with ω_2. If $AB = O_1A$, find all possible values of $\frac{r_1}{r_2}$.

5. In triangle ABC, $\angle A = 45°$ and M is the midpoint of $\overline{BC}$. $\overline{AM}$ intersects the circumcircle of ABC for the second time at D, and $AM = 2MD$. Find $\cos \angle AOD$, where O is the circumcenter of ABC.

6. Let $ABCD$ be a quadrilateral such that $\angle ABC = \angle CDA = 90°$, and $BC = 7$. Let E and F be on BD such that AE and CF are perpendicular to BD. Suppose that $BE = 3$. Determine the product of the smallest and largest possible lengths of DF.

7. Let ABC be an obtuse triangle with circumcenter O such that $\angle ABC = 15°$ and $\angle BAC > 90°$. Suppose that AO meets BC at D, and that $OD^2 + OC \cdot DC = OC^2$. Find $\angle C$.

8. Let $ABCD$ be a convex quadrilateral. Extend line CD past D to meet line AB at P and extend line CB past B to meet line AD at Q. Suppose that line AC bisects $\angle BAD$. If $AD = \frac{7}{4}$, $AP = \frac{21}{2}$, and $AB = \frac{14}{11}$, compute AQ.

9. Pentagon $ABCDE$ is given with the following conditions:

   (a) $\angle CBD + \angle DAE = \angle BAD = 45°$, $\angle BCD + \angle DEA = 300°$

   (b) $\frac{BA}{DA} = \frac{2\sqrt{2}}{3}$, $CD = \frac{7\sqrt{5}}{3}$, and $DE = \frac{15\sqrt{2}}{4}$

   (c) $AD^2 \cdot BC = AB \cdot AE \cdot BD$

   Compute BD.

10. Triangle ABC is inscribed in a circle ω. Let the bisector of angle A meet ω at D and BC at E. Let the reflections of A across D and C be D' and C', respectively. Suppose that $\angle A = 60°$, $AB = 3$, and $AE = 4$. If the tangent to ω at A meets line BC at P, and the circumcircle of APD' meets line BC at F (other than P), compute FC'.

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Geometry Test

PUT LABEL HERE

Name ____________________ Team ID# ________

Organization ____________________ Team ____________________

1. ____________
2. ____________
3. ____________
4. ____________
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9. ____________
10. ____________

Score: ____________