

HMMT November 2016

November 12, 2016

Theme Round

1. DeAndre Jordan shoots free throws that are worth 1 point each. He makes 40% of his shots. If he takes two shots find the probability that he scores at least 1 point.
2. Point P_1 is located 600 miles West of point P_2 . At 7:00 AM a car departs from P_1 and drives East at a speed of 50 miles per hour. At 8:00 AM another car departs from P_2 and drives West at a constant speed of x miles per hour. If the cars meet each other exactly halfway between P_1 and P_2 , what is the value of x ?
3. The three points A, B, C form a triangle. $AB = 4, BC = 5, AC = 6$. Let the angle bisector of $\angle A$ intersect side BC at D . Let the foot of the perpendicular from B to the angle bisector of $\angle A$ be E . Let the line through E parallel to AC meet BC at F . Compute DF .
4. A positive integer is written on each corner of a square such that numbers on opposite vertices are relatively prime while numbers on adjacent vertices are not relatively prime. What is the smallest possible value of the sum of these 4 numbers?
5. Steph Curry is playing the following game and he wins if he has exactly 5 points at some time. Flip a fair coin. If heads, shoot a 3-point shot which is worth 3 points. If tails, shoot a free throw which is worth 1 point. He makes $\frac{1}{2}$ of his 3-point shots and all of his free throws. Find the probability he will win the game. (Note he keeps flipping the coin until he has exactly 5 or goes over 5 points)
6. Let P_1, P_2, \dots, P_6 be points in the complex plane, which are also roots of the equation $x^6 + 6x^3 - 216 = 0$. Given that $P_1P_2P_3P_4P_5P_6$ is a convex hexagon, determine the area of this hexagon.
7. Seven lattice points form a convex heptagon with all sides having distinct lengths. Find the minimum possible value of the sum of the squares of the sides of the heptagon.
8. Let $P_1P_2 \dots P_8$ be a convex octagon. An integer i is chosen uniformly at random from 1 to 7, inclusive. For each vertex of the octagon, the line between that vertex and the vertex i vertices to the right is painted red. What is the expected number times two red lines intersect at a point that is not one of the vertices, given that no three diagonals are concurrent?
9. The vertices of a regular nonagon are colored such that 1) adjacent vertices are different colors and 2) if 3 vertices form an equilateral triangle, they are all different colors.
Let m be the minimum number of colors needed for a valid coloring, and n be the total number of colorings using m colors. Determine mn . (Assume each vertex is distinguishable.)
10. We have 10 points on a line $A_1, A_2 \dots A_{10}$ in that order. Initially there are n chips on point A_1 . Now we are allowed to perform two types of moves. Take two chips on A_i , remove them and place one chip on A_{i+1} , or take two chips on A_{i+1} , remove them, and place a chip on A_{i+2} and A_i . Find the minimum possible value of n such that it is possible to get a chip on A_{10} through a sequence of moves.