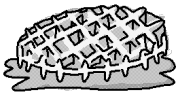
HMMT November 2019

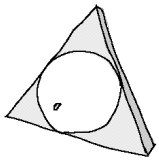
November 9, 2019

Theme Round

1. For breakfast, Mihir always eats a bowl of Lucky Charms cereal, which consists of oat pieces and marshmallow pieces. He defines the *luckiness* of a bowl of cereal to be the ratio of the number of marshmallow pieces to the total number of pieces. One day, Mihir notices that his breakfast cereal has exactly 90 oat pieces and 9 marshmallow pieces, and exclaims, "This is such an unlucky bowl!" How many marshmallow pieces does Mihir need to add to his bowl to double its luckiness?

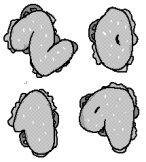
2. Sandy likes to eat waffles for breakfast. To make them, she centers a circle of waffle batter of radius 3cm at the origin of the coordinate plane and her waffle iron imprints non-overlapping unit-square holes **centered** at each lattice point. How many of these holes are contained entirely within the area of the waffle?





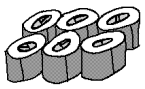
3. For breakfast, Milan is eating a piece of toast shaped like an equilateral triangle. On the piece of toast rests a single sesame seed that is one inch away from one side, two inches away from another side, and four inches away from the third side. He places a circular piece of cheese on top of the toast that is tangent to each side of the triangle. What is the area of this piece of cheese?

4. To celebrate 2019, Faraz gets four sandwiches shaped in the digits 2, 0, 1, and 9 at lunch. However, the four digits get reordered (but not flipped or rotated) on his plate and he notices that they form a 4-digit multiple of 7. What is the greatest possible number that could have been formed?



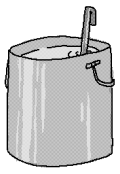
5. Alison is eating 2401 grains of rice for lunch. She eats the rice in a very peculiar manner: every step, if she has only one grain of rice remaining, she eats it. Otherwise, she finds the smallest positive integer $d > 1$ for which she can group the rice into equal groups of size d with none left over. She then groups the rice into groups of size d, eats one grain from each group, and puts the rice back into a single pile. How many steps does it take her to finish all her rice?

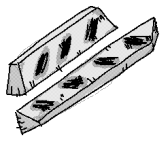
6. Wendy eats sushi for lunch. She wants to eat six pieces of sushi arranged in a 2×3 rectangular grid, but sushi is sticky, and Wendy can only eat a piece if it is adjacent to (not counting diagonally) at most two other pieces. In how many orders can Wendy eat the six pieces of sushi, assuming that the pieces of sushi are distinguishable?



7. Carl only eats food in the shape of equilateral pentagons. Unfortunately, for dinner he receives a piece of steak in the shape of an equilateral triangle. So that he can eat it, he cuts off two corners with straight cuts to form an equilateral pentagon. The set of possible perimeters of the pentagon he obtains is exactly the interval $[a, b)$, where a and b are positive real numbers. Compute $\frac{a}{b}$.

8. Omkar, Krit_1, Krit_2, and Krit_3 are sharing $x > 0$ pints of soup for dinner. Omkar always takes 1 pint of soup (unless the amount left is less than one pint, in which case he simply takes all the remaining soup). Krit_1 always takes $\frac{1}{6}$ of what is left, Krit_2 always takes $\frac{1}{5}$ of what is left, and Krit_3 always takes $\frac{1}{4}$ of what is left. They take soup in the order of Omkar, Krit_1, Krit_2, Krit_3, and then cycle through this order until no soup remains. Find all x for which everyone gets the same amount of soup.





9. For dinner, Priya is eating grilled pineapple spears. Each spear is in the shape of the quadrilateral $PINE$, with $PI = 6$cm, $IN = 15$cm, $NE = 6$cm, $EP = 25$cm, and $\angle NEP + \angle EPI = 60°$. What is the area of each spear, in cm^2?

10. For dessert, Melinda eats a spherical scoop of ice cream with diameter 2 inches. She prefers to eat her ice cream in cube-like shapes, however. She has a special machine which, given a sphere placed in space, cuts it through the planes $x = n$, $y = n$, and $z = n$ for every integer n (not necessarily positive). Melinda centers the scoop of ice cream uniformly at random inside the cube $0 \leq x, y, z \leq 1$, and then cuts it into pieces using her machine. What is the expected number of pieces she cuts the ice cream into?