Integration Bee Qualifiers

HMMT November 2022

November 11, 2022
8:00 - 9:00 pm

Instructions

- You will have **30 minutes** to complete **15 integration problems**, plus the **estimation tiebreaker**.
- Each integral, except the tiebreaker, is worth equal points.
- All logarithms are base e.
- You may omit the constant of integration (i.e., the $+C$) for indefinite integrals.
- You may leave your answers unsimplified, as long as they don't contain integrals, summations, or limits.

Do NOT turn over until you are told to begin.

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1 $\displaystyle\int_{-1}^{1} (x^3+1)^3\,dx$

2 $\displaystyle\int_{0}^{1} \frac{2x+3}{4x+5}\,dx$

3 $\displaystyle\int e^{e^x+x}\,dx$

4 $\displaystyle\int (\sin x)\,(\cos(e^{\cos x}))\,e^{\cos x}\,dx$

5 $\displaystyle\int \frac{1}{\sqrt{x+1}-\sqrt{x}}\,dx$

6 $\displaystyle\int_{0}^{15} \left\lfloor \frac{5x}{3} \right\rfloor dx$

7 $\displaystyle\int \frac{\log\log x}{x\log x}\,dx$

8 $\displaystyle\int \sin x\cos x\cdot(\sin x+\cos x)\,dx$

9 $\displaystyle\int_{0}^{2\pi} \lfloor 2022\sin x\rfloor\,dx$

10 $\displaystyle\int_{0}^{\pi/3} \frac{1}{(\cos x+\sqrt{3}\sin x)^2}\,dx$

11 $\displaystyle\int \frac{x^{20}}{(x+1)^{22}}\,dx$

12 $\displaystyle\int (x+1)e^x\log x\,dx$

13 $\displaystyle\int x\sin(\sqrt{1+x^2})\,dx$

14 $\displaystyle\int \frac{1}{\sqrt{1+\cos x}}\,dx$

15 $\displaystyle\int_{0}^{1} \frac{\log(1+x^7)}{x}\,dx$

16 Estimate the integral

$$\int_{0}^{1} e^{e^{e^x}}\,dx.$$

(For the tiebreaker, your answer must be an integer or terminating decimal. You will be scored by the **ratio** of your answer and the actual answer; that is, if you answer A and the answer is actually B, you score $\min(A/B, B/A)$.)

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Integration Bee Finals

HMMT November 2022

November 12, 2022

Format

- **2 minutes** per integral.
- We will check the result after two minutes.
- **Semifinal:** best of 3 integrals.
- **Final:** best of 5 integrals.
- If there is a tie, we will run a sudden death round.
- All logarithms are base e.
- You may omit the constant of integration (i.e., the $+C$) or the absolute values in the logarithms for indefinite integrals.
- You may leave your answers unsimplified, as long as they don't contain integrals, summations, or limits.

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Integration Bee Finals

HMMT November 2022

Sample

1 $\displaystyle\int_0^1 2022x\,dx$

2 $\displaystyle\int_0^1 x^{2022}\,dx$

Semifinal 1

1 $\displaystyle\int \frac{\log x}{x^{1+\log x}}\,dx$

2 $\displaystyle\int_0^{\frac{\pi}{2}} x(\sin x)^2\,dx$

3 $\displaystyle\int_0^1 (x^{11}+x)^9\,dx$

Semifinal 2

1 $\displaystyle\int_0^{\pi/3} \sqrt{\sin x \tan x}\,dx$

2 $\displaystyle\int_0^{10} x\{x\}\,dx$

3 $\displaystyle\int_1^2 \frac{x+1}{x^2+x\log x}\,dx$

Final

1 $\displaystyle\int \frac{x^2+2}{x^4+4}\,dx$

2 $\displaystyle\int_0^8 \sqrt{1+x^{-2/3}}\,dx$

3 $\displaystyle\int (x+1)e^{x(e^x+1)}\,dx$

4 $\displaystyle\int \frac{x(2\log x-1)}{(\log x)^2}\,dx$

5 $\displaystyle\int \frac{\arcsin x}{\sqrt{1+x}}\,dx$

Sudden Death

1 $\displaystyle\int_4^5 x(x-1)(x-2)(x-3)\left(\tfrac{1}{x}+\tfrac{1}{x-1}+\tfrac{1}{x-2}+\tfrac{1}{x-3}\right)\,dx$

2 $\displaystyle\int_0^{\pi/4} (\tan x)^3+\tan x\,dx$

3 $\displaystyle\int_{\sqrt{2}}^{\sqrt{3}} \frac{x}{x^4-1}\,dx$

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