# Integration Bee Qualifiers

## HMMT February 2023

**February 17, 2023**
**7:00 - 9:00 pm**

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## Instructions

- You will have **30 minutes** to complete **20 integration problems**.
- Each integral, except for the last question which serves as the tiebreaker, is worth equal points.
- All logarithms are base $e$.
- You may omit the constant of integration (i.e., the $+C$) or the absolute values in the logarithms for indefinite integrals.
- You may leave your answers unsimplified, as long as they don't contain integrals, summations, or limits.

**Do NOT turn over until you are told to begin.**

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# Integration Bee Qualifiers

## HMMT February 2023

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**1** $\displaystyle\int_{-1}^{1} (1+x)\left(1+x^{2023}\right)\,dx$

**2** $\displaystyle\int \frac{e^x+e}{e^x+ex+e}\,dx$

**3** $\displaystyle\int \frac{\sqrt[x]{2}}{x^2}\,dx$

**4** $\displaystyle\int \frac{2x^{2023}-x^{2022}}{x^{2024}-x^{2023}}\,dx$

**5** $\displaystyle\int \sqrt{x^{0!}\sqrt[3]{x^{1!}\sqrt[4]{x^{2!}\sqrt[5]{x^{3!}\cdots}}}}\,dx$

**6** $\displaystyle\int_{\pi/12}^{\pi/6} \frac{1}{\sin x\cos x\cos 2x}\,dx$

**7** $\displaystyle\int x^{(\log x)^{2023}-1}\cdot(\log x)^{2023}\,dx$

**8** $\displaystyle\int \frac{\cos x}{2-\cos^2 x}\,dx$

**9** $\displaystyle\int_{0}^{\pi/2} \frac{\cos^{2023} x}{\cos^{2023} x+\sin^{2023} x}\,dx$

**10** $\displaystyle\int \frac{1-3x}{\sqrt{1-x}}\,dx$

**11** $\displaystyle\int (x-1)e^{2/x}\,dx$

**12** $\displaystyle\int_{0}^{1} \left(\prod_{k=1}^{2023}(x+k)\right)\sum_{j=1}^{2023}\frac{1}{x+j}\,dx$

**13** $\displaystyle\int_{\pi/6}^{\pi/3} \frac{\sin^2 x}{x(\pi-2x)}\,dx$

**14** $\displaystyle\int \frac{1}{x\sqrt{x^{2023}-1}}\,dx$

**15** $\displaystyle\int_{-\infty}^{2023} \left\lceil \frac{x(1+\sin^2 x)}{1+x^2} \right\rceil dx$

**16** $\displaystyle\int \frac{1}{\sqrt{x^4-x^3}}\,dx$

**17** $\displaystyle\int \frac{1}{x\sec^2 x+\tan x}\,dx$

**18** $\displaystyle\int_{1}^{e} \left(x^{\log x}+e^{\sqrt{\log x}}\right)dx$

**19** $\displaystyle\int \sqrt{\frac{x+\sqrt{1+x^2}}{1+x^2}}\,dx$

**20** Estimate the integral

$$\int_{1}^{e^4} x^{\log x}\,dx$$

(For the tiebreaker, if you answer $A$ and the answer is actually $B$, your score will be $\min(A/B, B/A)$. **You may submit your guess unevaluated**, like $\pi^{10}/2023$.)

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# Integration Bee Finals

## HMMT February 2023

**February 18, 2023**

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### Format

- **2 minutes** per integral.
- We will check the result after two minutes.
- **Semifinal:** best of 3 integrals.
- **Final:** best of 5 integrals.
- If there is a tie, we will run a sudden death round.
- All logarithms are base $e$.
- You may omit the constant of integration (i.e., the $+C$) or the absolute values in the logarithms for indefinite integrals.
- You may leave your answers unsimplified, as long as they don't contain integrals, summations, or limits.

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# Integration Bee Finals

HMMT February 2023

## Sample

**1** $\displaystyle\int_0^1 2023x\,dx$

**2** $\displaystyle\int_0^1 x^{2023}\,dx$

## Semifinal 1

**1** $\displaystyle\int_0^{2023!} \{x\} + \left\{\frac{x}{2}\right\} + \left\{\frac{x}{3}\right\} + \cdots + \left\{\frac{x}{2023}\right\}\,dx$

**2** $\displaystyle\int \frac{\log\left(x^2+1\right)}{x^2}\,dx$

**3** $\displaystyle\int_0^3 \sqrt{\sqrt{x+1}-1}\,dx$

## Semifinal 2

**1** $\displaystyle\int \frac{\sin^3 x}{\cos^4 x}\,dx$

**2** $\displaystyle\int e^{e^x}e^x\left(xe^x+x+1\right)dx$

**3** $\displaystyle\int \frac{\arcsin x}{\sqrt{1+x}}\,dx$

## Third Place

**1** $\displaystyle\int_0^{2023} \min\left(3x, \frac{3}{2}\left(2023-x\right)\right)\,dx$

**2** $\displaystyle\int \frac{1}{x^{3/4}+x^{5/4}}\,dx$

**3** $\displaystyle\int_0^{2\pi} (\sin x + 3\cos x)^4\,dx$

## Final

**1** $\displaystyle\int \frac{\log(\sin x)}{\cos^2 x}\,dx$

**2** $\displaystyle\int x^{(\log x)^2-2}(3(\log x)^2-1)\,dx$

**3** $\displaystyle\int_0^1 \sin\left(\frac{\arcsin x}{3}\right)\,dx$

**4** $\displaystyle\int \frac{1}{(x\tan x+1)^2}\,dx$

**5** $\displaystyle\int_0^1 \frac{3x-4x^3}{\sqrt{1-x^2}}\,dx$

## Sudden Death

**1** $\displaystyle\int_0^1 \frac{(1-2x)e^{x-x^2}}{\sqrt{x-x^2}}\,dx$

**2** $\displaystyle\int_0^{100\pi} x\lfloor\sin x\rfloor\,dx$

**3** $\displaystyle\int \cos 3x\cos^2 x\,dx$

**4** $\displaystyle\int \frac{x^2}{x^4-1}\,dx$

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