Integration Bee Qualifiers

HMMT November 2023

November 10, 2023
7:00 - 9:00 pm

Instructions

- You will have **30 minutes** to complete **20 integration problems**.
- Each integral, except for the last question which serves as the tiebreaker, is worth equal points.
- All logarithms are base e.
- You may omit the constant of integration (i.e., the $+C$) or the absolute values in the logarithms for indefinite integrals.
- You may leave your answers unsimplified, as long as they don't contain integrals, summations, or limits.

Do NOT turn over until you are told to begin.

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1 $\displaystyle\int_0^{\sqrt[2024]{2024}} 2023x^{2023}\,dx$

2 $\displaystyle\int_0^1 \frac{1+x+x^2}{1+x}\,dx$

3 $\displaystyle\int_0^{\pi} \sin x \cos x \tan x\,dx$

4 $\displaystyle\int_0^{1/2} \sqrt{1-4x^2}\,dx$

5 $\displaystyle\int \left(x^2+\frac{1}{x^2}\right)^2\,dx$

6 $\displaystyle\int_0^1 x\sqrt{x^2+1}\,dx$

7 $\displaystyle\int \sin^8 x \sin 2x\,dx$

8 $\displaystyle\int \frac{1}{\sqrt{x+20}+\sqrt{x+23}}\,dx$

9 $\displaystyle\int (x^2+2x-1)\sqrt{x+1}\,dx$

10 $\displaystyle\int \frac{\tan x+\tan 1}{1-\tan x\cdot\tan 1}\,dx$

11 $\displaystyle\int \frac{x}{\sqrt{1-x^2}}\,dx$

12 $\displaystyle\int_0^{2023} \left(\frac{1}{1+e^{\sin x}}+\frac{1}{1+e^{-\sin x}}\right)\,dx$

13 $\displaystyle\int_0^{2023} \left\{\frac{x}{17}\right\}\,dx$

14 $\displaystyle\int_0^1 \ln\left(1+x+x^2+x^3+\cdots\right)\,dx$

15 $\displaystyle\int \frac{\arctan x}{1+x^2}\,dx$

16 $\displaystyle\int \frac{1}{1+2023e^x}\,dx$

17 $\displaystyle\int \frac{\cos x}{5-\cos^2 x}\,dx$

18 $\displaystyle\int_0^{20} x\lceil \sin \pi x\rceil\,dx$

19 $\displaystyle\int_0^1 \sum_{k=1}^{2023} \frac{1}{x+k}\,dx$

20 Estimate

$$\ln\left(\int_1^{e^{e^e}} (\ln x)^{\ln x}dx\right)$$

(For the tiebreaker, if you answer A and the answer is actually B, your score will be $\min(A/B, B/A)$. **You may submit your guess unevaluated**, like $\pi^{10}/2023$.)

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Integration Bee Finals

HMMT November 2023

November 11, 2023

Format

- **2 minutes** per integral.
- We will check the result after two minutes.
- **Semifinal:** best of 3 integrals.
- **Final:** best of 5 integrals.
- If there is a tie, we will run a sudden death round.
- All logarithms are base e.
- You may omit the constant of integration (i.e., the $+C$) or the absolute values in the logarithms for indefinite integrals.
- You may leave your answers unsimplified, as long as they don't contain integrals, summations, or limits.

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Integration Bee Finals

HMMT November 2023

Sample

1 $\displaystyle\int_0^1 2023x\,dx$

2 $\displaystyle\int_0^1 x^{2023}\,dx$

Semifinal 1

1 $\displaystyle\int \frac{(x+1)^2}{x^2+1}\,dx$

2 $\displaystyle\int \ln(x^3+3x^2+3x+1)\,dx$

3 $\displaystyle\int_{-1}^1 \min(\arctan x, x^2)+\max(\arctan x, x^2)\,dx$

Semifinal 2

1 $\displaystyle\int_1^e \frac{\ln(ex)}{x}\,dx$

2 $\displaystyle\int_0^{\pi/4} \frac{1+\tan^2 x}{1+\tan x}\,dx$

3 $\displaystyle\int x^3(1-x)^4 - x^4(1-x)^3\,dx$

Third Place

1 $\displaystyle\int_0^1 \frac{x^4+x^2+1}{x^2-x+1}\,dx$

2 $\displaystyle\int \cos(2\arccos x)\,dx$

3 $\displaystyle\int (\ln x)^2(\ln(x)+3)dx$

Final

1 $\displaystyle\int_0^1 2^{\log_3 x}+3^{\log_2 x}\,dx$

2 $\displaystyle\int \frac{\cos(2x)}{\cos^2 x}\,dx$

3 $\displaystyle\int \frac{1+\ln x}{x^x}\,dx$

4 $\displaystyle\int_0^\infty \frac{1}{e^x+1}\,dx$

5 $\displaystyle\int_0^2 \sqrt{x+\sqrt{x+\sqrt{x+...}}}dx$

Sudden Death

1 $\displaystyle\int_0^1 \lfloor x^2+x\rfloor\,dx$

2 $\displaystyle\int_{-2}^4 \max(x, \lfloor x\rfloor)\,dx$

3 $\displaystyle\int_0^\infty e^{-\sqrt{x}}\,dx$

4 $\displaystyle\int_1^\infty \frac{\ln(1+x)}{x^2}\,dx$

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