HMMT November 2023

November 11, 2023

Theme Round

1. Tyler has an infinite geometric series with sum 10. He increases the first term of his sequence by 4 and swiftly changes the subsequent terms so that the common ratio remains the same, creating a new geometric series with sum 15. Compute the common ratio of Tyler's series.

2. Suppose rectangle $FOLK$ and square $LORE$ are on the plane such that $RL = 12$ and $RK = 11$. Compute the product of all possible areas of triangle RKL.

3. There are 17 people at a party, and each has a reputation that is either 1, 2, 3, 4, or 5. Some of them split into pairs under the condition that within each pair, the two people's reputations differ by at most 1. Compute the largest value of k such that no matter what the reputations of these people are, they are able to form k pairs.

4. Let $LOVER$ be a convex pentagon such that $LOVE$ is a rectangle. Given that $OV = 20$ and $LO = VE = RE = RL = 23$, compute the radius of the circle passing through R, O, and V.

5. Compute the unique positive integer n such that $\frac{n^3-1989}{n}$ is a perfect square.

6. A function g is *ever more* than a function h if, for all real numbers x, we have $g(x) \geq h(x)$. Consider all quadratic functions $f(x)$ such that $f(1) = 16$ and $f(x)$ is ever more than both $(x+3)^2$ and x^2+9. Across all such quadratic functions f, compute the minimum value of $f(0)$.

7. Betty has a 3×4 grid of dots. She colors each dot either red or maroon. Compute the number of ways Betty can color the grid such that there is no rectangle whose sides are parallel to the grid lines and whose vertices all have the same color.

8. Call a number *feared* if it contains the digits 13 as a contiguous substring and *fearless* otherwise. (For example, 132 is feared, while 123 is fearless.) Compute the smallest positive integer n such that there exists a positive integer $a < 100$ such that n and $n + 10a$ are fearless while $n + a, n + 2a, \ldots, n + 9a$ are all feared.

9. Pentagon $SPEAK$ is inscribed in triangle NOW such that S and P lie on segment NO, K and A lie on segment NW, and E lies on segment OW. Suppose that $NS = SP = PO$ and $NK = KA = AW$. Given that $EP = EK = 5$ and $EA = ES = 6$, compute OW.

10. It is midnight on April 29th, and Abigail is listening to a song by her favorite artist while staring at her clock, which has an hour, minute, and second hand. These hands move continuously. Between two consecutive midnights, compute the number of times the hour, minute, and second hands form two equal angles and no two hands overlap.