Integration Bee Qualifiers

HMMT February 2024

February 16, 2024
7:00 - 9:00 PM

Instructions

- You will have **30 minutes** to complete **20 integration problems**.
- Each integral, except for the last question which serves as the tiebreaker, is worth equal points.
- All logarithms are base e.
- You may omit the constant of integration (i.e., the $+C$) or the absolute values in the logarithms for indefinite integrals.
- You may leave your answers unsimplified, as long as they don't contain integrals, summations, or limits.

Do NOT turn over until you are told to begin.

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1 $\displaystyle\int_0^1 d'd'\,dd'$

2 $\displaystyle\int_1^5 \frac{x}{\lfloor x \rfloor}\,dx$

3 $\displaystyle\int_0^1 \ln(1+\sqrt{x})\,dx$

4 $\displaystyle\int 2^{\ln x}\,dx$

5 $\displaystyle\int e^{e^x+2x}+e^{e^x+x}\,dx$

6 $\displaystyle\int_0^{2024} x^{\log_{x^2}(2024)}\,dx$

7 $\displaystyle\int_{-2}^{2} \arctan(2^{\sinh(x)})\,dx$

8 $\displaystyle\int x\cot^2(x)\,dx$

9 $\displaystyle\int \frac{1}{x^{4/3}+x^{5/3}}\,dx$

10 $\displaystyle\int_0^{\pi/3} \frac{\sin(3x)}{\sin(2x)-\sin(x)}\,dx$

11 $\displaystyle\int_{4/3}^{\infty} \frac{\sqrt{1+x^2}}{x^4}\,dx$

12 $\displaystyle\int_0^1 4\sin(x+\sqrt{x})\sin(x-\sqrt{x})\,dx$

13 $\displaystyle\int_0^{\infty} \frac{1}{x^3+x^2+x+1}\,dx$

14 $\displaystyle\int \frac{\cos(x)+2\sin(x)}{\cos(x)+\sin(x)\cos^2(x)}\,dx$

15 $\displaystyle\int_0^{\infty} \frac{\sqrt{x}-1}{x^2-\sqrt{x}}\,dx$

16 $\displaystyle\int_0^{\infty} \frac{x^2+1}{x^4+1}\,dx$

17 $\displaystyle\int_0^1 \arccos(x)\ln(x)\,dx$

18 $\displaystyle\int_1^2 \sqrt{x-\sqrt{x^2-1}}\,dx$

19 $\displaystyle\int_0^{2\pi} \ln\left(\sin(x)+\tfrac{5}{4}\right)\,dx$

20 Estimate

$$\int_{-\infty}^{\infty} e^{x^3-e^x}\,dx.$$

(For the tiebreaker, if you answer A and the answer is actually B, your score will be $\min(A/B, B/A)$. **You may submit your guess unevaluated**, like $\pi^{10}/2023$.)

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Integration Bee Finals

HMMT February 2024

February 17, 2024

Format

- **2 minutes** per integral.
- We will check the result after two minutes.
- **Semifinal:** best of 3 integrals.
- **Final:** best of 5 integrals.
- If there is a tie, we will run a sudden death round.
- All logarithms are base e.
- You may omit the constant of integration (i.e., the $+C$) or the absolute values in the logarithms for indefinite integrals.
- You may leave your answers unsimplified, as long as they don't contain integrals, summations, or limits.

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Integration Bee Finals

HMMT February 2024

Sample

1 $\displaystyle\int_0^1 2024x\,dx$

2 $\displaystyle\int_0^1 x^{2024}\,dx$

Semifinal 1

1 $\displaystyle\int_0^1 \sin x \sin(1-x)\,dx$

2 $\displaystyle\int_0^1 \sin(\ln x)\,dx$

3 $\displaystyle\int_0^{\sqrt{\pi}} \arctan(\cot(x^2))\,dx$

Semifinal 2

1 $\displaystyle\int \sqrt{\sec x}\tan x\,dx$

2 $\displaystyle\int_0^{\pi} \ln(\sin(x))\,dx$

3 $\displaystyle\int_0^{\infty} \frac{\arctan(x)}{x^2+2x+1}\,dx$

Third Place

1 $\displaystyle\int_0^1 \ln(1-x^2)\,dx$

2 $\displaystyle\int \frac{x}{\tan x+\cot x}\,dx$

3 $\displaystyle\int_0^{\frac{\pi}{2}} \sin(x)\sin(2x)\sin(3x)\,dx$

Final

1 $\displaystyle\int \frac{\ln(1+x)}{x\sqrt{x}}\,dx$

2 $\displaystyle\int_{-\infty}^{\infty} \frac{\sin x}{x(\pi-x)}\,dx$

3 $\displaystyle\int_{-1}^{1} \sin(\arcsin(x^2)-\arcsin(x))\,dx$

4 $\displaystyle\int_0^e \frac{\ln x+(\ln x)^2}{x^{x-1}}\,dx$

5 $\displaystyle\int \sqrt{x^2-\frac{5}{4}+\sqrt{(x+1)^2-\frac{5}{4}+\sqrt{(x+2)^2-\frac{5}{4}+\cdots}}}\,dx$

Sudden Death

1 $\displaystyle\int_0^3 \lfloor x\rfloor^x\,dx$

2 $\displaystyle\int_0^{\infty} xe^{-\sqrt{x}}\,dx$

3 $\displaystyle\int_0^{\pi} \cos^6(x)\,dx$

4 $\displaystyle\int \frac{\ln x}{(x+1)^2}\,dx$

5 $\displaystyle\int \frac{x^2+2x\sin(x)}{1+\cos(x)}\,dx$

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