Integration Bee Qualifiers

HMMT November 2024

November 8, 2024
7:00 - 9:00 PM

Instructions

- You will have **30 minutes** to complete **20 integration problems**.
- Each integral, except for the last question which serves as the tiebreaker, is worth equal points.
- All logarithms are base e.
- You may omit the constant of integration (i.e., the $+C$) or the absolute values in the logarithms for indefinite integrals.
- You may leave your answers unsimplified, as long as they don't contain integrals, summations, or limits.

Do NOT turn over until you are told to begin.

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1 $\displaystyle\int_0^{4048} |x - 2024|\, dx$

2 $\displaystyle\int_0^{2024} \lfloor \lceil x \rceil \rfloor - \lceil \lfloor x \rfloor \rceil\, dx$

3 $\displaystyle\int_0^{\pi/2} \sqrt{\sin(x)\cos(x)\tan(x)}\, dx$

4 $\displaystyle\int \frac{2^x}{1^x + 4^x}\, dx$

5 $\displaystyle\int (x-1)^{100}(x+1)^{101} + (x+1)^{100}(x-1)^{101}\, dx$

6 $\displaystyle\int \tan(\tan x)(1 + \tan^2 x)\, dx$

7 $\displaystyle\int_1^{e^\pi} \sin(\log(x))\, dx$

8 $\displaystyle\int_0^1 \log(1 + x^2)\, dx$

9 $\displaystyle\int_0^{\pi/2} \max\left(\sqrt{\sin x \cdot \cos x}, \frac{\sin x + \cos x}{2}\right) dx$

10 $\displaystyle\int_0^{\pi} \sin(x)\sin^2(2x)\, dx$

11 $\displaystyle\int \frac{1}{\sqrt[3]{x+1} - \sqrt[3]{x}} - \sqrt[3]{x(x+1)}\, dx$

12 $\displaystyle\int_{-\infty}^0 \left(\left(2^{2^0}\right)^x + \left(2^{2^1}\right)^x + \left(2^{2^2}\right)^x + \ldots\right) dx$

13 $\displaystyle\int \log(x\log x - x)\log x\, dx$

14 $\displaystyle\int_0^\infty \frac{\arctan x}{(1+x)\sqrt{x}}\, dx$

15 $\displaystyle\int_{-1}^0 \min(x^2 - 1, x^3) + \max(x^2, x^3 + 1)\, dx$

16 $\displaystyle\int \frac{x^2 + 2}{(x^2 - 2)^2}\, dx$

17 $\displaystyle\int \left(\frac{e^x - e^{-x}}{e^x + e^{-x}}\right)^2 dx$

18 $\displaystyle\int_1^\infty \frac{\lfloor\sqrt{x}\rfloor}{x^2}\, dx$

19 $\displaystyle\int \frac{e^x(1 + \sin x)}{1 + \cos x}\, dx$

20 Estimate

$$\int_0^{2\pi} e^{\sin x}\, dx.$$

(For the tiebreaker, if you answer A and the answer is actually B, your score will be $\min(A/B, B/A)$. **You may submit your guess unevaluated**, like $\pi^{10}/2024$.)

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Integration Bee Finals

HMMT November 2024

November 9, 2024

Format

- **2 minutes** per integral.
- We will check the result after two minutes.
- **Semifinal:** best of 3 integrals.
- **Final:** best of 5 integrals.
- If there is a tie, we will run a sudden death round.
- All logarithms are base e.
- You may omit the constant of integration (i.e., the $+C$) or the absolute values in the logarithms for indefinite integrals.
- You may leave your answers unsimplified, as long as they don't contain integrals, summations, or limits.

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Integration Bee Finals

HMMT November 2024

Sample

1 $\displaystyle\int_0^1 2024x\,dx$

2 $\displaystyle\int_0^1 x^{2024}\,dx$

Semifinal 1

1 $\displaystyle\int x\sec^2 x\,dx$

2 $\displaystyle\int_0^1 \frac{1}{\sqrt{x}+\sqrt[6]{x}}\,dx$

3 $\displaystyle\int_0^\infty \frac{(24x^{26}-25x^{25}+x)e^{-x}}{(x-1)^2}\,dx$

Semifinal 2

1 $\displaystyle\int \frac{1}{x\sqrt{x+1}+(x+1)\sqrt{x}}\,dx$

2 $\displaystyle\int_0^1 \sqrt{\sqrt{x}+1}\,dx$

3 $\displaystyle\int_0^1 \arcsin x \arccos x\,dx$

Audience Round

1 $\displaystyle\int_{-1.5}^{1.5} \lfloor \tan x \rfloor\,dx$

Final

1 $\displaystyle\int_1^\infty \frac{(x-\lfloor x\rfloor)^{\lfloor x\rfloor}}{\lfloor x\rfloor}\,dx$

2 $\displaystyle\int_0^1 x^{x^{x+1}}+\log\left(x^{x^{x^{x+1}}}\right)\,dx$

3 $\displaystyle\int_6^{60} \sqrt[3]{x+\sqrt[3]{x+\sqrt[3]{x+\ldots}}}\,dx$

4 $\displaystyle\int_1^\infty x^{-1-\sqrt{1/\log x}}\,dx$

5 $\displaystyle\int_0^{2\pi} \sqrt{(1-\sin x)(1+\cos x)}\,dx$

Sudden Death

1 $\displaystyle\int x^{\log x}(2\log x+1)\,dx$

2 $\displaystyle\int_{-\infty}^\infty |4x+1|-|3x+2|-|2x+3|+|x+4|\,dx$

3 $\displaystyle\int \sqrt{\log x}\,(2\log x+3)\,dx$

4 $\displaystyle\int_0^{\pi^2} \sin\sqrt{x}\,dx$

5 $\displaystyle\int_{-\infty}^0 2^{2^x}\cdot 4^x\,dx$

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