Integration Bee Qualifiers

HMMT February 2025

February 14, 2025
7:00 - 9:00 PM

Instructions

- You will have **30 minutes** to complete **20 integration problems**.
- Each integral, except for the last question which serves as the tiebreaker, is worth equal points.
- All logarithms are base e unless otherwise specified.
- You may omit the constant of integration (i.e., the $+C$) or the absolute values in the logarithms for indefinite integrals.
- You may leave your answers unsimplified, as long as they don't contain integrals, summations, or limits.

Do NOT turn over until you are told to begin.

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1 $\displaystyle\int_{-45}^{45} |x|\,dx$

2 $\displaystyle\int_{0}^{2025} \lfloor x+1 \rfloor - \lceil x \rceil\,dx$

3 $\displaystyle\int_{-1}^{\sqrt{3}} \sqrt{4-x^2}\,dx$

4 $\displaystyle\int 2^{\log_3 x}\,dx$

5 $\displaystyle\int 2^{x+2^x}\,dx$

6 $\displaystyle\int_{-\infty}^{\infty} \frac{1}{e^x+e^{-x}}\,dx$

7 $\displaystyle\int \frac{x^2}{x^2+2x+2}\,dx$

8 $\displaystyle\int \frac{1+\log x}{x^x}\,dx$

9 $\displaystyle\int \frac{\tan^2 x}{\tan x - x}\,dx$

10 $\displaystyle\int \frac{x}{x+\sqrt{x^2+1}}\,dx$

11 $\displaystyle\int \frac{\log(1+e^x)}{e^x}\,dx$

12 $\displaystyle\int_{-1}^{1} (\arccos(x))^2 - (\arcsin(x))^2\,dx$

13 $\displaystyle\int_{0}^{1} x^{x^{x^{x^{\cdots}}}} + x^{1/x}\,dx$

14 $\displaystyle\int_{-\infty}^{\infty} \frac{\lfloor \sin x \rfloor}{x^2+1}\,dx$

15 $\displaystyle\int x^{\log x - 1}\log x\,dx$

16 $\displaystyle\int_{-\infty}^{\infty} \frac{x^2+1}{x^4+x^2+1}\,dx$

17 $\displaystyle\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \log(\sin x+\cos x)\,dx$

18 $\displaystyle\int \tan(x)\tan(2x)\tan(3x)\,dx$

19 $\displaystyle\int_{0}^{\frac{\pi}{2}} \tan(x)\log(\sin x)\,dx$

20 Estimate

$$\int_0^1 \prod_{n=1}^{20}(1+x^n)\,dx.$$

(For the tiebreaker, if you answer a positive real number A and the answer is actually B, your score will be $\min(A/B, B/A)$. Otherwise, your score will be 0. **You may submit your guess unevaluated**, like $\pi^{10}/2025$.)

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Integration Bee Finals

HMMT February 2025

February 15, 2025

Format

- **2 minutes** per integral.
- We will check the result after two minutes.
- **Semifinal:** best of 3 integrals.
- **Final:** best of 5 integrals.
- If there is a tie, we will run a sudden death round.
- All logarithms are base e.
- You may omit the constant of integration (i.e., the $+C$) or the absolute values in the logarithms for indefinite integrals.
- You may leave your answers unsimplified, as long as they don't contain integrals, summations, or limits.

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Integration Bee Finals

HMMT February 2025

Sample

1 $\displaystyle\int_0^1 2025x\,dx$

2 $\displaystyle\int_0^1 x^{2025}\,dx$

Semifinal 1

1 $\displaystyle\int \arctan\sqrt{x}\,dx$

2 $\displaystyle\int_0^{2025} \frac{\sqrt{x+\sqrt{x+\sqrt{x+\ldots}}}}{1+\frac{x}{1+\frac{x}{1+\ldots}}}\,dx$

3 $\displaystyle\int_0^1 \frac{\sqrt{1-x^2}}{1+x}\,dx$

Semifinal 2

1 $\displaystyle\int \frac{x-1}{x+e^x}\,dx$

2 $\displaystyle\int \frac{\log\left(x^x(1+x)^{1+x}\right)}{x(1+x)}\,dx$

3 $\displaystyle\int_0^1 (1-\log x)^4\,dx$

Audience Round

1 $\displaystyle\int_{-\infty}^{\infty} \frac{\lfloor x\rfloor}{e^x+e^{-x}}\,dx$

Final

1 $\displaystyle\int_0^1 x^{20}\log\left(x^{25}\right)\,dx$

2 $\displaystyle\int_{-1/2}^{1/2} \frac{1}{1+x^{2025}}-\frac{1}{1-x^{4050}}\,dx$

3 $\displaystyle\int_0^1 \arcsin\left(2x\sqrt{1-x^2}\right)\,dx$

4 $\displaystyle\int \frac{\sin 2x}{\sin x+\cos x}\,dx$

5 $\displaystyle\int_0^{2\pi} \sqrt{\sqrt{2}-\sin x-\cos x}\,dx$

Sudden Death

1 $\displaystyle\int_{-1}^1 \sin\left(\cos^{-1}\left(\tan\left(\csc^{-1}\left(\sec\left(\cot^{-1}(x)\right)\right)\right)\right)\right)\,dx$

2 $\displaystyle\int \frac{1+\log(x)}{1+x^2\log^2(x)}\,dx$

3 $\displaystyle\int \frac{\log x}{x^2}\,dx$

4 $\displaystyle\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \cos(x)\sqrt{\cos(2x)}\,dx$

5 $\displaystyle\int x^{2025}\sin\left(x^{1013}\right)\,dx$

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