HMMT November 2025

November 08, 2025

General Round

1. Let $ABCD$ be a rectangle. Let X and Y be points on segments $\overline{BC}$ and $\overline{AD}$, respectively, such that $\angle AXY = \angle XYC = 90^\circ$. Given that $AX : XY : YC = 1 : 2 : 1$ and $AB = 1$, compute BC.

2. Suppose n integers are placed in a circle such that each of the following conditions is satisfied:
   - at least one of the integers is 0;
   - each pair of adjacent integers differs by exactly 1; and
   - the sum of the integers is exactly 24.

   Compute the smallest value of n for which this is possible.

3. Ashley fills each cell of a 3×3 grid with some of the numbers 1, 2, 3, and 4 (possibly none or several). Compute the number of ways she can do so such that each row and each column contains each of 1, 2, 3, and 4 exactly once. (One such grid is shown below.)

1 2	3 4	
4	1	2 3
3	2	1 4

4. Given that a, b, and c are integers with $c \leq 2025$ such that $|x^2 + ax + b| = c$ has exactly 3 distinct integer solutions for x, compute the number of possible values of c.

5. Let A, B, C, and D be points on a line in that order. There exists a point E such that $\angle AED = 120^\circ$ and triangle BEC is equilateral. Given that $BC = 10$ and $AD = 39$, compute $|AB - CD|$.

6. Kelvin the frog is at the point $(0, 0, 0)$ and wishes to reach the point $(3, 3, 3)$. In a single move, he can either increase any single coordinate by 1, or he can decrease his z-coordinate by 1. Given that he cannot visit any point twice, and that at all times his coordinates must all stay between 0 and 3 (inclusive), compute the number of distinct paths Kelvin can take to reach $(3, 3, 3)$.

7. A positive integer n is *imbalanced* if strictly more than 99 percent of the positive divisors of n are strictly less than 1 percent of n. Given that M is an imbalanced multiple of 2000, compute the minimum possible number of positive divisors of M.

8. Let Γ_1 and Γ_2 be two circles that intersect at two points P and Q. Let ℓ_1 and ℓ_2 be the common external tangents of Γ_1 and Γ_2. Let Γ_1 touch ℓ_1 and ℓ_2 at U_1 and U_2, respectively, and let Γ_2 touch ℓ_1 and ℓ_2 at V_1 and V_2, respectively. Given that $PQ = 10$ and the distances from P to ℓ_1 and ℓ_2 are 3 and 12, respectively, compute the area of the quadrilateral $U_1U_2V_2V_1$.

9. Let a, b, and c be pairwise distinct nonzero complex numbers such that

$$(10a + b)(10a + c) = a + \tfrac{1}{a},$$
$$(10b + a)(10b + c) = b + \tfrac{1}{b},$$
$$(10c + a)(10c + b) = c + \tfrac{1}{c}.$$

   Compute abc.

10. Jacob and Bojac each start in a cell of the same 8×8 grid (possibly different cells). They listen to the same sequence of cardinal directions (North, South, East, and West). When a direction is called out, Jacob always walks one cell in that direction, while Bojac always walks one cell in the direction 90° counterclockwise of the called direction. If either person cannot make their move without leaving the grid, that person stays still instead. Over all possible starting positions and sequences of instructions, compute the maximum possible number of distinct ordered pairs (Jacob's position, Bojac's position) that they could have reached.