

HMMT November 2025

November 08, 2025

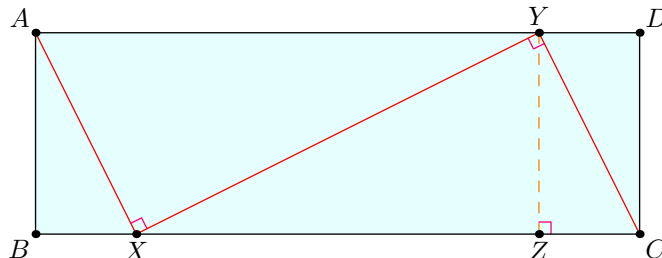
General Round

1. Let $ABCD$ be a rectangle. Let X and Y be points on segments $\overline{BC}$ and $\overline{AD}$, respectively, such that $\angle AXY = \angle XYC = 90^\circ$. Given that $AX : XY : YC = 1 : 2 : 1$ and $AB = 1$, compute BC .

Proposed by: Derek Liu, Marin Hristov Hristov

Answer: 3

Solution:



Let Z be the foot from Y to BC . Then, $\triangle ABX \sim \triangle XZY$. Since $XY/AX = 2$, we know $XZ = 2AB = 2$ and $BX = ZY/2 = 1/2$. Similarly, $CZ = 1/2$, so $BC = 1/2 + 2 + 1/2 = \boxed{3}$.

2. Suppose n integers are placed in a circle such that each of the following conditions is satisfied:

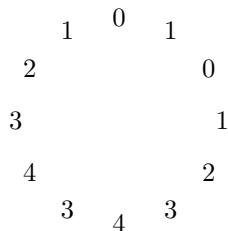
- at least one of the integers is 0;
- each pair of adjacent integers differs by exactly 1; and
- the sum of the integers is exactly 24.

Compute the smallest value of n for which this is possible.

Proposed by: Derek Liu

Answer: 12

Solution: Note n is always even because the numbers in the circle alternate parity. If $n \leq 8$, then the sum is at most $0 + 1 + 2 + 3 + 4 + 3 + 2 + 1 = 16$, so $n \geq 10$. If $n = 10$, the sum consists of 5 even and 5 odd numbers, so it cannot be even. Thus, $n \geq \boxed{12}$, achieved with the following diagram.



3. Ashley fills each cell of a 3×3 grid with some of the numbers 1, 2, 3, and 4 (possibly none or several). Compute the number of ways she can do so such that each row and each column contains each of 1, 2, 3, and 4 exactly once. (One such grid is shown below.)

12	34	
4	1	23
3	2	14

Proposed by: Ashley Zhu

Answer: 1296

Solution: We consider the placements of each of 1, 2, 3, and 4 separately. Observe that there are $3! = 6$ ways to place the 1's, as each row must have a 1 and they must be in different columns. The same holds for 2, 3, and 4. Thus, the answer is $6^4 = \boxed{1296}$.

4. Given that a , b , and c are integers with $c \leq 2025$ such that $|x^2 + ax + b| = c$ has exactly 3 distinct integer solutions for x , compute the number of possible values of c .

Proposed by: Srinivas Arun

Answer: 31

Solution: The answer is 31. The possible values of c are $c = 2k^2$ for integers $1 \leq k \leq 31$. These work by taking $a = 0$ and $b = -2k^2$; then $|x^2 - 2k^2| = 2k^2$ is equivalent to $x^2 = 0$ or $4k^2$, so it has three integer solutions: $x = 0, -2k, 2k$.

We will now prove no other values of c work. The problem condition is equivalent to the two quadratic equations

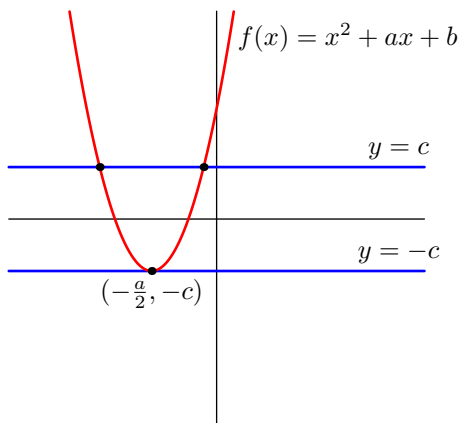
$$x^2 + ax + b = c \quad \text{and} \quad x^2 + ax + b = -c$$

having three distinct integer solutions in total.

If $c = 0$, then these two quadratics are the same, so they have at most two roots in total, contradiction. Hence $c \neq 0$, so the two quadratics share no roots.

If one of these quadratics has an integer root, then both its roots are integers. This follows from Vieta's formulas: for each quadratic, the sum of the roots is $-a$, an integer. If only one quadratic has integer roots, then the two quadratics have at most two integer roots in total, contradiction. So both quadratics have all integer roots.

Because the two quadratics have three distinct integer roots in total, and all roots of both quadratics are integers, the two quadratics have three distinct roots in total. Since the quadratics share no roots, one quadratic has two roots and the other has one root. Graphically, this means that the vertex of $x^2 + ax + b$ lies on either the line $y = c$ or $y = -c$. Since this parabola faces upward, the case $y = c$ is impossible (since then $x^2 + ax + b$ never intersects $y = -c$), so the vertex lies on $y = -c$. Refer to the diagram below.



Because $x^2 + ax + b = -c$ has one root, $x^2 + ax + (b + c)$ is a perfect square; it must be $(x + \frac{a}{2})^2$, and so $x^2 + ax + (b + c) = 0$ has the single solution $x = -\frac{a}{2}$. This implies $\frac{a}{2}$ is an integer.

Next we use the condition that $x^2 + ax + b = c$ has integer roots. Complete the square to obtain that

$$x^2 + ax + b = c \implies x^2 + ax + (b + c) = \left(x + \frac{a}{2}\right)^2 = 2c.$$

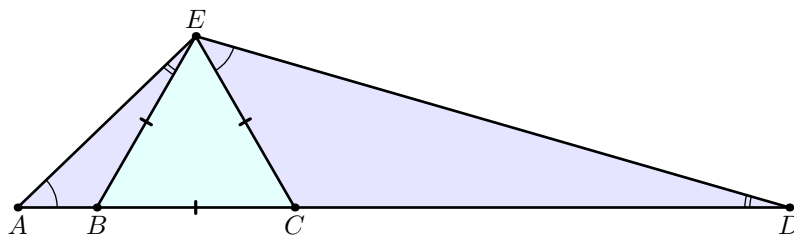
Since $\frac{a}{2}$ and x are both integers, $2c$ must be a (nonzero) perfect square. Note that $2c$ is even, so let $2c = (2k)^2$ for some integer k . This rewrites as $c = 2k^2$. Since $1 \leq c \leq 2025$, $1 \leq k^2 \leq 1012$, so $1 \leq k \leq 31$. Hence these are the only values of c that work, as desired.

5. Let A, B, C , and D be points on a line in that order. There exists a point E such that $\angle AED = 120^\circ$ and triangle BEC is equilateral. Given that $BC = 10$ and $AD = 39$, compute $|AB - CD|$.

Proposed by: Jackson Dryg

Answer: 21

Solution:



First, we note that $\angle ABE = \angle AED = 120^\circ$, so $\triangle ABE \sim \triangle AED$. Similarly, $\triangle ECD \sim \triangle AED$, so $\triangle ABE \sim \triangle ECD$. Now, let $AB = x$ and $CD = y$. Then,

$$\frac{x}{10} = \frac{10}{y} \implies xy = 100.$$

Furthermore, $x + y = AD - BC = 29$, so

$$(x - y)^2 = (x + y)^2 - 4xy = 29^2 - 4 \cdot 100 = 441.$$

Hence, the answer is $|x - y| = \sqrt{441} = \boxed{21}$.

6. Kelvin the frog is at the point $(0,0,0)$ and wishes to reach the point $(3,3,3)$. In a single move, he can either increase any single coordinate by 1, or he can decrease his z -coordinate by 1. Given that he cannot visit any point twice, and that at all times his coordinates must all stay between 0 and 3 (inclusive), compute the number of distinct paths Kelvin can take to reach $(3,3,3)$.

Proposed by: Derek Liu

Answer: $4^6 \binom{6}{3} = 81920$

Solution: Projecting the path down to the xy -plane, we get an up-right path from $(0,0)$ to $(3,3)$, of which there are $\binom{6}{3} = 20$. Fix such an up-right path. Each edge in this path can be the projection of four different edges Kelvin can take (at $z = 0, 1, 2$, and 3), so there are 4^6 ways to pick the six edges that project down to the up-right path. These can be connected into a unique path with vertical moves, so the answer is $4^6 \binom{6}{3} = \boxed{81920}$.

7. A positive integer n is *imbalanced* if strictly more than 99 percent of the positive divisors of n are strictly less than 1 percent of n . Given that M is an imbalanced multiple of 2000, compute the minimum possible number of positive divisors of M .

Proposed by: Srinivas Arun

Answer: 1305

Solution: Note that M has at least 13 divisors which are at least $\frac{M}{100}$, namely

$$\begin{aligned} \frac{M}{1}, \frac{M}{2}, \frac{M}{4}, \frac{M}{8}, \frac{M}{16}, \\ \frac{M}{5}, \frac{M}{10}, \frac{M}{20}, \frac{M}{40}, \frac{M}{80}, \\ \frac{M}{25}, \frac{M}{50}, \frac{M}{100}. \end{aligned}$$

Therefore, since more than 99 percent of M 's divisors are less than $\frac{M}{100}$, we know M must have at least $13 \cdot 100 + 1 = 1301$ divisors in total. Consider $M = 2^4 \cdot 5^{260}$. Note that M has $(4+1)(260+1) = \boxed{1305}$ divisors and the only divisors of M that are at least $\frac{M}{100}$ are the 13 divisors listed above. We now show that the number of positive divisors of M can't be between 1301 and 1304 inclusive.

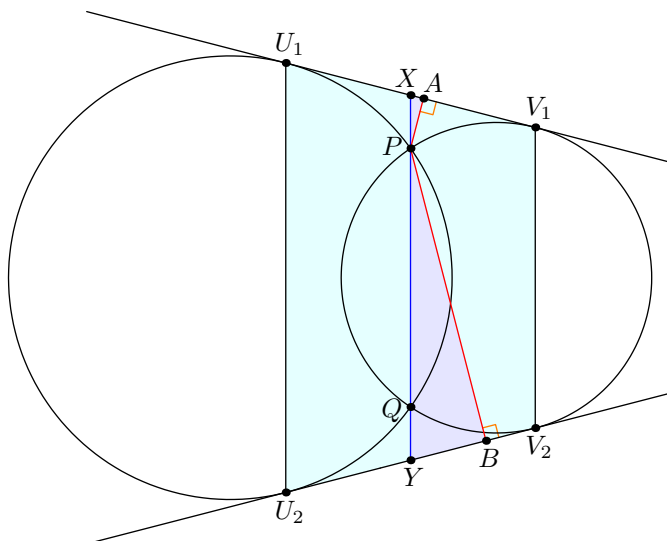
Observe that $32 \nmid M$. Indeed, if $32 \mid M$, then $\frac{M}{32}$ would be a divisor of M , meaning there would be at least 14 divisors that are at least $\frac{M}{100}$. This would mean M has at least $14 \cdot 100 + 1 = 1401$ divisors, which is not minimal due to the construction above achieving 1305 divisors. As $2000 \mid M$ and $32 \nmid M$, we get that $\nu_2(M) = 4$. In particular, this means the number of divisors of M is divisible by $4 + 1 = 5$, so M can't have between 1301 and 1304 divisors inclusive. This concludes the solution.

8. Let Γ_1 and Γ_2 be two circles that intersect at two points P and Q . Let ℓ_1 and ℓ_2 be the common external tangents of Γ_1 and Γ_2 . Let Γ_1 touch ℓ_1 and ℓ_2 at U_1 and U_2 , respectively, and let Γ_2 touch ℓ_1 and ℓ_2 at V_1 and V_2 , respectively. Given that $PQ = 10$ and the distances from P to ℓ_1 and ℓ_2 are 3 and 12, respectively, compute the area of the quadrilateral $U_1U_2V_2V_1$.

Proposed by: Aprameya Tripathy

Answer: 200

Solution:



Let $\overline{PQ}$ meet ℓ_1 and ℓ_2 at X and Y , respectively, and let the feet from P to ℓ_1 and ℓ_2 be A and B , respectively. By symmetry, note that $U_1U_2V_2V_1$ and U_2U_1XY are isosceles trapezoids. We will find both the height and the midline length of $U_1U_2V_2V_1$ to compute its area.

Since U_2U_1XY is an isosceles trapezoid, we have that $\angle PXA = \angle PYB$. Furthermore, $\angle PAX = 90^\circ = \angle PBY$, so $\triangle PXA \sim \triangle PYB$. Then, since $QY = PX$ by symmetry,

$$\frac{PQ}{XP} = \frac{PY}{XP} - 1 = \frac{PB}{PA} - 1 = \frac{12}{3} - 1 = 3,$$

meaning $XP = \frac{10}{3}$ and $XQ = XP + PQ = \frac{40}{3}$. Thus, $\sin \angle PXA = \frac{PA}{XP} = \frac{9}{10}$. Since lines PQ and U_1U_2 are parallel, we have that $\angle PXA = \angle U_2U_1V_1$, so the height of trapezoid $U_1U_2V_2V_1$ is $U_1V_1 \cdot \sin \angle PXA = \frac{9}{10}U_1V_1$. Now, by Power of a Point,

$$XU_1^2 = XP \cdot XQ = XV_1^2,$$

and

$$YU_2^2 = YP \cdot YQ = YV_2^2$$

so X and Y are the midpoints of U_1V_1 and U_2V_2 , respectively, and $U_1V_1 = 2\sqrt{XP \cdot XQ} = \frac{40}{3}$. Therefore, XY is the midline of $U_1U_2V_2V_1$ and has length $PQ + 2 \cdot PX = 10 + 2 \cdot \frac{20}{3} = \frac{50}{3}$, while the height of $U_1U_2V_2V_1$ is $\frac{9}{10}U_1V_1 = 12$.

Thus, the area of trapezoid $U_1U_2V_2V_1$ is $12 \cdot \frac{50}{3} = \boxed{200}$.

9. Let a , b , and c be pairwise distinct nonzero complex numbers such that

$$\begin{aligned}(10a + b)(10a + c) &= a + \frac{1}{a}, \\ (10b + a)(10b + c) &= b + \frac{1}{b}, \\ (10c + a)(10c + b) &= c + \frac{1}{c}.\end{aligned}$$

Compute abc .

Proposed by: Pitchayut Saengrungrongka, Qiao Zhang

Answer: $\boxed{\frac{1}{91}}$

Solution 1: Consider the polynomial $f(x) = (x + a)(x + b)(x + c)$. The given conditions can be rewritten as

$$\begin{aligned}f(10a) &= (10a + a)(a + \frac{1}{a}) = 11(a^2 + 1), \\ f(10b) &= (10b + b)(b + \frac{1}{b}) = 11(b^2 + 1), \\ f(10c) &= (10c + c)(c + \frac{1}{c}) = 11(c^2 + 1).\end{aligned}$$

Thus, $10a$, $10b$, and $10c$ are roots to $f(x) - 11(\frac{x^2}{100} + 1)$. Since a , b , and c are pairwise distinct, these three roots are distinct, so

$$(x + a)(x + b)(x + c) = (x - 10a)(x - 10b)(x - 10c) + 11\left(\frac{x^2}{100} + 1\right).$$

Comparing the constant term of both sides gives

$$abc = -1000abc + 11 \implies abc = \frac{11}{1001} = \boxed{\frac{1}{91}}.$$

Remark. By comparing coefficients of the equations above, we get $a + b + c = \frac{1}{100}$ and $ab + bc + ca = 0$. Together with $abc = \frac{1}{91}$, this solves (a, b, c) up to permutation and shows that such (a, b, c) exist.

Solution 2: Subtracting the second equation from the first yields

$$100(a^2 - b^2) + 9ac - 9bc = a - b + \frac{1}{a} - \frac{1}{b}.$$

As a and b are distinct, we can divide both sides by $a - b$ to find

$$100(a + b) + 9c = 1 - \frac{1}{ab}.$$

A similar argument shows

$$100(b + c) + 9a = 1 - \frac{1}{bc}.$$

The difference of these two equations gives us

$$100(a - c) + 9(c - a) = \frac{1}{bc} - \frac{1}{ab} \implies 91(a - c) = \frac{a - c}{abc}.$$

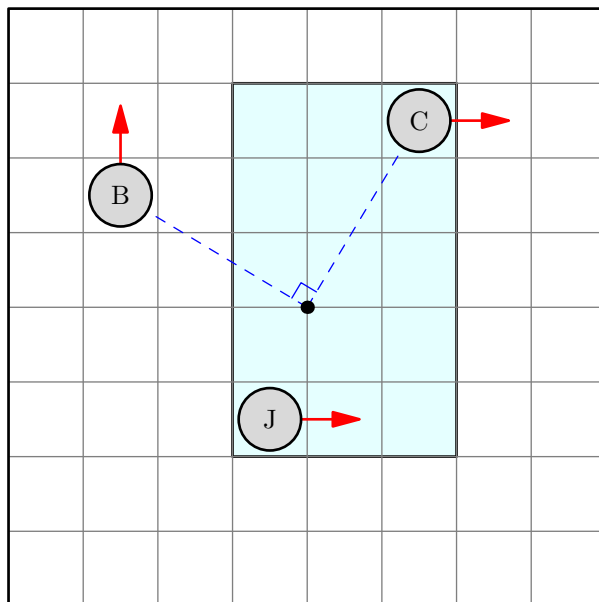
Since a and c are distinct, we can divide both sides by $a - c$ to get $abc = \boxed{\frac{1}{91}}$.

10. Jacob and Bojac each start in a cell of the same 8×8 grid (possibly different cells). They listen to the same sequence of cardinal directions (North, South, East, and West). When a direction is called out, Jacob always walks one cell in that direction, while Bojac always walks one cell in the direction 90° counterclockwise of the called direction. If either person cannot make their move without leaving the grid, that person stays still instead. Over all possible starting positions and sequences of instructions, compute the maximum possible number of distinct ordered pairs (Jacob's position, Bojac's position) that they could have reached.

Proposed by: Sebastian Attlan

Answer: $\boxed{372}$

Solution:



Suppose there is a third person, Caboj, who starts at the cell which is a 90° clockwise rotation of Bojac's position about the grid's center, and always moves in the called direction (unless doing so would leave the grid). Notice that Caboj's path is precisely the 90° clockwise rotation of Bojac's path about the center. Therefore, to maximize the number of ordered pairs (Jacob's position, Bojac's position), it suffices to maximize the number of ordered pairs (Jacob's position, Caboj's position).

Consider the (possibly degenerate) rectangle with opposite vertices at Jacob's and Caboj's positions. When Jacob and Caboj both move, this rectangle gets translated by 1 unit in some direction, and when only one of Jacob and Caboj moves, this rectangle shrinks by 1 unit in some dimension. In particular, both side lengths of this rectangle, and hence the taxicab distance between Jacob and Caboj, are nonincreasing.

Suppose this taxicab distance is currently d , and the rectangle has dimensions $x \times y$, where $x + y = d + 2$. There are $(9 - x)(9 - y)$ distinct translations of this rectangle within the grid, all of which are achievable without shrinking the rectangle. Therefore, there are at most $(9 - x)(9 - y)$ ordered pairs of positions with the rectangle having size $x \times y$. This quantity is maximized when x and y are as close as possible, i.e., $x = y = d/2 + 1$ when d is even and $\{x, y\} = \{(d + 1)/2, (d + 3)/2\}$ when d is odd.

Since the side lengths of the rectangle cannot increase, only one rectangle size can be reached for each taxicab distance d . Summing over all possible taxicab distances, we get an upper bound of

$$\begin{aligned} & (1)(1) + (1)(2) + (2)(2) + (2)(3) + (3)(3) + (3)(4) + \cdots + (7)(8) + (8)(8) \\ &= 2(1^2 + 2^2 + \cdots + 7^2) + (1 + 2 + \cdots + 7) + 8^2 \\ &= 2\left(\frac{7 \cdot 8 \cdot 15}{6}\right) + \frac{7 \cdot 8}{2} + 64 \\ &= \boxed{372}. \end{aligned}$$

We can construct this by first having a 8×8 rectangle and achieving $1 \cdot 1$ position, then shrinking to a 7×8 rectangle and achieving $1 \cdot 2$ positions, then shrinking to a 7×7 rectangle and achieving $2 \cdot 2$ positions, and so on (alternating which dimension we shrink), until we have a 1×1 rectangle and achieve $8 \cdot 8$ positions.