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1. **[5]** Compute the remainder when 1000! is divided by 1001.

2. **[5]** Sebastian fills the four squares in $\square^{\square^{\square^{\square}}}$ with the numbers 2, 0, 2, and 5, in some order. Compute the sum of all distinct possible values of the resulting expression.

   (Here, $a^{b^{c^d}}$ is evaluated as $a^{(b^{(c^d)})}$. For instance, $2^{2^{2^2}} = 2^{2^4} = 2^{16} = 65536$.)

3. **[5]** Square $ABCD$ has side length 45. Points W, X, Y, and Z lie on sides $\overline{AB}$, $\overline{BC}$, $\overline{CD}$, and $\overline{DA}$, respectively, such that $AW = CY = 20$ and $BX = DZ = 25$. Compute the area of quadrilateral $WXYZ$.

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4. **[6]** A mercury thermometer reads the temperature using three temperature scales: $^\circ C$ (Celsius), $^\circ F$ (Fahrenheit), and $^\circ S$ (Saengrungkongka). The conversions are as follows:

   - A temperature of $x^\circ C$ corresponds to $(\frac{9}{5}x + 32)^\circ F$.
   - A temperature of $x^\circ S$ corresponds to $(\frac{9}{5}x + 32)^\circ C$.

   Given that the current temperature readings on all three scales are odd positive integers, compute the minimum possible value of the current temperature in degrees **Celsius**.

5. **[6]** A circle of radius strictly less than 2 is drawn in the plane. Compute the maximum possible number of lattice points that can lie on its circumference. (A lattice point is a point with integer coordinates.)

6. **[6]** Compute the number of ways to color each cell of an 8×8 grid either red, green, or blue such that every 1×3 and 3×1 rectangle with edges on the grid lines contains exactly one cell of each color.

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7. **[7]** Point X lies on diagonal $\overline{AC}$ of rectangle $ABCD$ such that $AX = 11$, $CX = 1$, and triangle BXD has area 18. Given that $BX < DX$, compute BX.

8. **[7]** Compute the sum of the distinct prime factors of 20202525.

9. **[7]** Suppose S and T are two sets of distinct positive integers, each with 15 elements, such that S and T have no elements in common. Further suppose

$$\operatorname{sum}(S) = \operatorname{sum}(T) = k,$$

where $\operatorname{sum}(A)$ denotes the sum of the elements of A. Compute the minimum possible value of k.

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10. **[8]** Compute the number of positive divisors of 10^{20} that leave a remainder of 1 when divided by 9.

11. **[8]** Jessica has a non-square rectangular sheet of paper with all 4 corners colored differently. She repeats the following process 8 times: she picks one of the rectangle's two axes of symmetry, then flips the rectangle over that axis. Compute the number of ways she can do this so that each corner ends up in its original position.

12. **[8]** Let $ABCD$ be a right trapezoid such that $\angle ABC = \angle BCD = 90^\circ$ and the circle with diameter $\overline{AD}$ is tangent to side $\overline{BC}$. Given that $AB = 7$ and $BC = 8$, compute CD.

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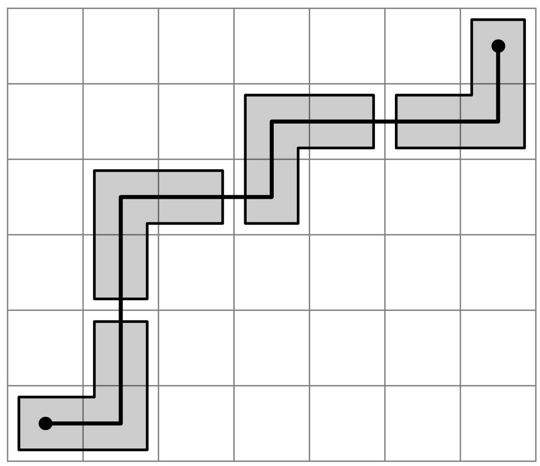
13. **[9]** Let P be a point and ℓ be a line in the coordinate plane.

- If point P were reflected across ℓ and then translated by $(+0, +6)$, the result would be point A.
- If point P were translated by $(+0, +6)$ and then reflected across ℓ, the result would be point B.

Given that $AB = 10$, compute the maximum possible area of triangle PAB.

14. **[9]** Marin starts on the bottom-left square of a 6×7 grid and walks to the top-right square by taking steps one square either up or to the right. Given that the set of squares Marin visits on his walk can be partitioned into L-trominoes, compute the number of ways that Marin can complete his walk.

An *L-tromino* is a set of three squares formed by removing exactly one square from a 2×2 grid of squares. One example of a valid path is shown below:



15. **[9]** Compute

$$\sum_{k=1}^{\infty} \frac{1}{2^{2^k} - 2^{-2^k}} = \frac{1}{2^{2^1} - 2^{-2^1}} + \frac{1}{2^{2^2} - 2^{-2^2}} + \frac{1}{2^{2^3} - 2^{-2^3}} + \cdots .$$

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16. **[10]** Let a_1, a_2, a_3, a_4, and a_5 be the five distinct complex solutions of $x^5 - 20x + 25 = 0$. Compute $a_1^4 + a_2^4 + a_3^4 + a_4^4 + a_5^4$.

17. **[10]** Let P be a point inside equilateral triangle ABC such that $\angle BPC = 150^\circ$. Given that circumradii of triangle ABP and triangle ACP are 3 and 5, respectively, compute AP.

18. **[10]** Compute the number of ways to divide a 6×6 square into 36 triangles, each of which has side lengths $\sqrt{2}$, $\sqrt{2}$, and 2. (Rotations and reflections of a division are considered distinct divisions.)

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19. **[11]** Compute the number of ordered triples of positive integers (a, b, c) such that b is a divisor of 2025 and $\frac{a}{b} + \frac{b}{c} = \frac{a}{c}$.

20. **[11]** Suppose that $ABCD$ and $AXYZ$ are squares with side lengths 10 and 7, respectively. Given that X lies inside triangle ABY and Y lies on segment $\overline{BD}$, compute the area of triangle BXC.

21. **[11]** Sarunyu starts at a vertex of a regular 7-gon. At each step, he chooses an unvisited vertex uniformly at random and walks to it along a straight line. He continues until all vertices are visited, and then walks back to his starting vertex along a straight line. A self-intersection occurs when two of his steps cross strictly inside the 7-gon. Compute the expected number of self-intersections in Sarunyu's walk.

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22. **[12]** Suppose that a, b, and c are pairwise distinct nonzero complex numbers such that

$$a^3 - 4a^2 + 5bc = b^3 - 4b^2 + 5ac = c^3 - 4c^2 + 5ab = 67.$$

Compute abc.

23. **[12]** Jacopo and Srinivas are playing a game with a bag of marbles. The bag starts with 6 red marbles and 6 blue marbles. Jacopo begins by drawing a marble from the bag, uniformly at random. When either player draws a marble, if it is red, the same player draws the next marble; otherwise, the other player draws the next marble (uniformly at random). All marbles are drawn without replacement. This process continues until all 12 marbles have been drawn. Compute the expected number of marbles that Jacopo draws.

24. **[12]** Let $ABCDE$ be a convex pentagon such that $ABCD$ is a rectangle and $\angle AEB = \angle CED = 30°$. Given that $AB = 14$ and $BC = 20\sqrt{3}$, compute the area of triangle ADE.

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25. **[13]** Jacob finished a rather hectic 9 holes of golf: he scored a 1 on the first hole, a 2 on the second hole, and so on, ending with a 9 on the ninth hole. Each hole is assigned a par of 3, 4, or 5. For each hole, Jacob subtracts its par from his score on that hole. Given that these 9 (possibly negative) differences can be arranged to form a sequence of 9 consecutive integers, compute the number of ways the pars could have been assigned to the holes.

26. **[13]** Rectangles $ABXP$ and $CDXQ$ lie inside semicircle $\mathcal{S}$ such that A, B, C, and D lie on the arc of $\mathcal{S}$, and P and Q lie on the diameter of $\mathcal{S}$. Given that $BX = 7$, $PX = 6$, and $QX = 8$, compute DX.

27. **[13]** Let a_1, a_2, a_3, $\ldots$ be a sequence of integers such that $a_1 = 2$ and $a_{n+1} = a_n^7 - a_n + 1$ for all $n \geq 1$. Compute the remainder when a_{500} is divided by 7^3.

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28. **[15]** Let S be the set of integers between 0 and 100 (inclusive). Compute the number of ways to color each element of S either red, green, or blue such that for all elements x and y of S with $|x-y|-1$ divisible by 3, the colors of x and y are different.

29. **[15]** Compute the smallest positive integer multiple of 10001 with all of its digits distinct (when written in base 10).

30. **[15]** Point P lies inside triangle ABC such that $BP = PC$ and $\angle APC - \angle APB = 60^\circ$. Given that $AP = 12$, $AB = 20$, and $AC = 25$, compute BC.

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31. **[17]** Gumdrops come in 7 different colors. Mark has two boxes of gumdrops, each containing one gumdrop of each color. He repeats the following process 7 times: he removes one gumdrop uniformly at random from each box, then eats one of the two removed gumdrops uniformly at random and throws away the other. Compute the probability Mark eats one gumdrop of each color.

32. **[17]** Compute the smallest positive integer n for which n^n (written in base 10) ends in 123.

33. **[17]** Four points A, B, C, and D lie on a circle with radius 2 such that $CD = 3$, $CA = CB$, and $DA - DB = 1$. Compute the maximum possible value of AB.

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34. **[20]** Estimate the number of integers in $\{1, 2, 3, \ldots, 10^8\}$ that can be written in the form $x^2 - 2025y^2$ for some integers x and y.

    Submit a positive integer E. If the correct answer is A, you will receive $\left\lfloor 20.99 \max\left(0, 1 - \frac{|E-A|}{A}\right)^{2.5} \right\rfloor$ points.

35. **[20]** Derek has a 45×45 grid of cells. Initially, every cell in the grid is uncolored. Every second, he picks one of the remaining uncolored cells uniformly at random and colors it in. He stops once there exist two distinct colored cells that share an edge. Estimate the expected number of seconds before Derek stops.

    Submit a real number E. If the correct answer is A, you will receive $\lceil \max(0, 20 - 7|E - A|^{3/4}) \rceil$ points.

36. **[20]** For each of the following properties, compute the smallest positive perfect square n with that property. (Unless otherwise specified, assume all numbers are written in base 10.)

    1. n contains at least 4 of the same nonzero digit.
    2. n starts with the same 2-digit number repeated twice (i.e., $\underline{abab}$ for digits a and b).
    3. $n > 2025$ contains 2025 as a contiguous substring.
    4. n contains 2520 as a contiguous substring.
    5. The sum of the digits of n is 25.
    6. When n is written in base 2025, the sum of its digits is 2025. (Submit n in base 10.)
    7. $n > 10000$, and the last four digits of n are also a perfect square (possibly with leading 0s).
    8. n^2 has 2025 positive divisors.
    9. n has two positive divisors that sum to 2025.

    Submit an 9-tuple of integers corresponding to the answers above, or an X for any value you wish to leave blank. For instance, if you think the first and last answers are 2025, you should submit "2025, X, X, X, X, X, X, X, 2025". If you submit C values and they are **all** correct, then you will receive $\lfloor C^2/4 \rfloor$ points. Otherwise, you will receive 0 points.