HMMT November 2025

November 08, 2025

Theme Round

- 1. Mark has two one-liter flasks: flask A and flask B. Initially, flask A is fully filled with liquid mercury, and flask B is partially filled with liquid gallium. Mark pours the contents of flask A into flask B until flask B is full. Then, he mixes the contents of flask B and pours it back into flask A until flask A is full again. Given that the mixture in flask B is now 30% mercury, and the mixture in flask A is X^{∞} mercury, compute X.
- 2. Uranus has 29 known moons. Each moon is blue, icy, or large, though some moons may have several of these characteristics. There are 10 moons which are blue but not icy, 8 moons which are icy but not large, and 6 moons which are large but not blue. Compute the number of moons which are simultaneously blue, icy, and large.
- 3. Let VENUS be a convex pentagon with area 84. Given that NV is parallel to SU, SE is parallel to UN, and triangle SUN has area 24, compute the maximum possible area of triangle EUV.
- 4. Compute the unique 5-digit integer <u>EARTH</u> for which the following addition holds:

The digits E, A, R, T, and H are not necessarily distinct, but the leading digits E and H must be nonzero.

5. Compute the number of ways to erase 26 letters from the string

SUNSUNSUNSUNSUNSUNSUNSUNSUN

such that the remaining 4 letters spell SUNS in order.

- 6. Regular hexagon SATURN (with vertices in counterclockwise order) has side length 2. Point O is the reflection of T over S. Hexagon SATURN is rotated 45° counterclockwise around O. Compute the area its interior traces out during this rotation.
- 7. Io, Europa, and Ganymede are three of Jupiter's moons. In one Jupiter month, they complete exactly I, E, and G orbits around Jupiter, respectively, for some positive integers I, E, and G. Each moon appears as a full moon precisely at the start of each of its orbits. Suppose that in every Jupiter month, there are
 - exactly 54 moments of time with at least one full moon,
 - exactly 11 moments of time with at least two full moons, and
 - at least 1 moment of time with all three full moons.

Compute $I \cdot E \cdot G$.

- 8. Let MARS be a trapezoid with \overline{MA} parallel to \overline{RS} and side lengths MA = 11, AR = 17, RS = 22, and SM = 16. Point X lies on side \overline{MA} such that the common chord of the circumcircles of triangles MXS and AXR bisects segment \overline{RS} . Compute MX.
- 9. Triton performs an ancient Neptunian ritual consisting of drawing red, green, and blue marbles from a bag. Initially, Triton has 3 marbles of each color, and the bag contains an additional 3 marbles of each color. Every turn, Triton picks one marble to put into the bag, then draws one marble uniformly at random from the bag (possibly the one he just discarded). The ritual is completed once Triton has 6 marbles of one color and 3 of another. Compute the expected number of turns the ritual will take, given that Triton plays optimally to minimize this value.

10. The orbits of Pluto and Charon are given by the ellipses

$$x^2 + xy + y^2 = 20$$
 and $2x^2 - xy + y^2 = 25$,

respectively. These orbits intersect at four points that form a parallelogram. Compute the largest of the slopes of the four sides of this parallelogram.