HMMT November 2025

November 08, 2025

Theme Round

1. Mark has two one-liter flasks: flask A and flask B. Initially, flask A is fully filled with liquid mercury, and flask B is partially filled with liquid gallium. Mark pours the contents of flask A into flask B until flask B is full. Then, he mixes the contents of flask B and pours it back into flask A until flask A is full again. Given that the mixture in flask B is now 30% mercury, and the mixture in flask A is x% mercury, compute x.

   Proposed by: Derek Liu

   Answer: $\boxed{79}$

   Solution: Since flask B is now 30% mercury, it must also have been 30% mercury (and hence 70% gallium) when it was full. For this to occur, Mark must have initially poured 0.3 liters of mercury from flask A into flask B. Thus, Mark also poured 0.3 liters of mixture from flask B into flask A, so flask A ended up with $0.3 \cdot (70\%) = 0.21$ liters of gallium. The remaining $\boxed{79}$% is mercury.

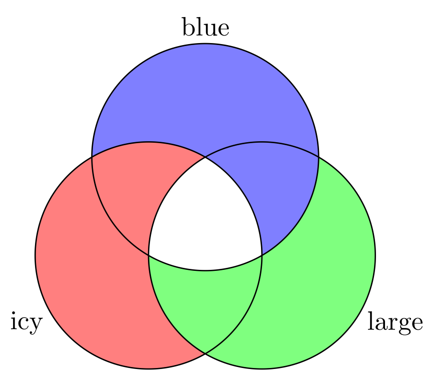
   Remark. Answers 79% and 0.79 instead of 79 were also accepted.

2. Uranus has 29 known moons. Each moon is blue, icy, or large, though some moons may have several of these characteristics. There are 10 moons which are blue but not icy, 8 moons which are icy but not large, and 6 moons which are large but not blue. Compute the number of moons which are simultaneously blue, icy, and large.

   Proposed by: Derek Liu

   Answer: $\boxed{5}$

   Solution:



   Consider the Venn diagram above.

   - The blue region represents moons which are blue but not icy.
   - The red region represents moons which are icy but not large.
   - The green region represents moons which are large but not blue.

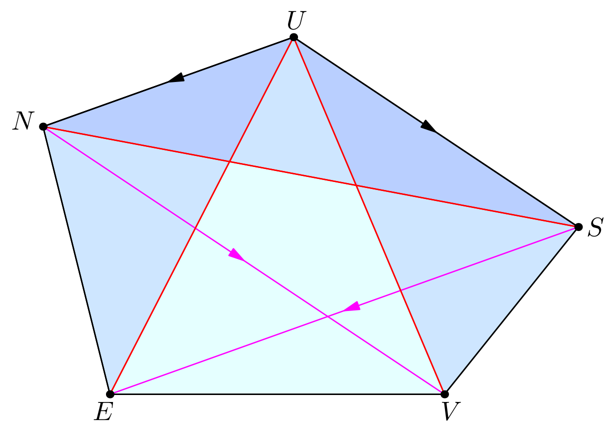
   Every moon is in at least one of the three circles, so the uncolored region is precisely the moons which are blue, icy, and large. Thus, the number of such moons is $29 - 10 - 8 - 6 = \boxed{5}$.

3. Let $VENUS$ be a convex pentagon with area 84. Given that NV is parallel to SU, SE is parallel to UN, and triangle SUN has area 24, compute the maximum possible area of triangle EUV.

   Proposed by: Alan Vladimiroff, David Wei

   Answer: $\boxed{36}$

   Solution:



   As SE is parallel to UN, triangles SUN and EUN have the same height from base UN. Therefore, $[EUN] = [SUN] = 24$. Similarly, since NV is parallel to SU, triangles SUN and SUV have the same height from base SU. Therefore, $[SUV] = [SUN] = 24$. Finally, we compute

$$[EUV] = [VENUS] - [EUN] - [SUV] = 84 - 24 - 24 = \boxed{36}.$$

4. Compute the unique 5-digit integer $\underline{EARTH}$ for which the following addition holds:

$$\begin{array}{cccccc} & H & A & T & E & R \\ + & H & E & A & R & T \\ \hline & E & A & R & T & H \end{array}$$

   The digits E, A, R, T, and H are not necessarily distinct, but the leading digits E and H must be nonzero.

   Proposed by: Derek Liu

   Answer: $\boxed{99774}$

   Solution: Looking at the thousands place, we see $E = 0$ or 9. The former is impossible by the given condition, so $E = 9$. Looking at the ten thousands place, we get $H = 4$.

   Looking at the ones place, $R + T$ is either 4 or 14, so R and T are of the same parity. Looking at the tens place, either $R \equiv T \pmod{10}$, or $R \equiv T + 1 \pmod{10}$. Thus, $R = T = 2$, or $R = T = 7$. The former is impossible because the tens place addition requires a carry, so $R = T = 7$. The hundreds place then leads to $A = 9$, so $\underline{EARTH} = \boxed{99774}$, which can be checked directly to satisfy the addition.

5. Compute the number of ways to erase 26 letters from the string

   SUNSUNSUNSUNSUNSUNSUNSUNSUNSUN

   such that the remaining 4 letters spell SUNS in order.

Proposed by: Derek Liu

Answer: $\boxed{\binom{12}{4} = 495}$

Solution: The remaining letters divide the 26 erased letters into five (possibly empty) strings, with the first four having length congruent to 0 mod 3 and the last having length congruent to 2 mod 3. See the example below; remaining letters are boxed, and the five strings have lengths 3, 6, 0, 15, and 2.

SUN**S**UNSUNS**UN**SUNSUNSUNSUNSUN**S**UN

If we write the lengths of these five strings as as $3a_0$, $3a_1$, $3a_2$, $3a_3$, and $3a_4 + 2$, then their sum is equal to 26, the number of erased letters. Hence, $3(a_0 + a_1 + a_2 + a_3 + a_4) + 2 = 26$, so we seek the number of nonnegative integer solutions to $a_0 + a_1 + a_2 + a_3 + a_4 = (26 - 2)/3 = 8$. By stars and bars, the answer is $\binom{8+4}{4} = \binom{12}{4} = \boxed{495}$.

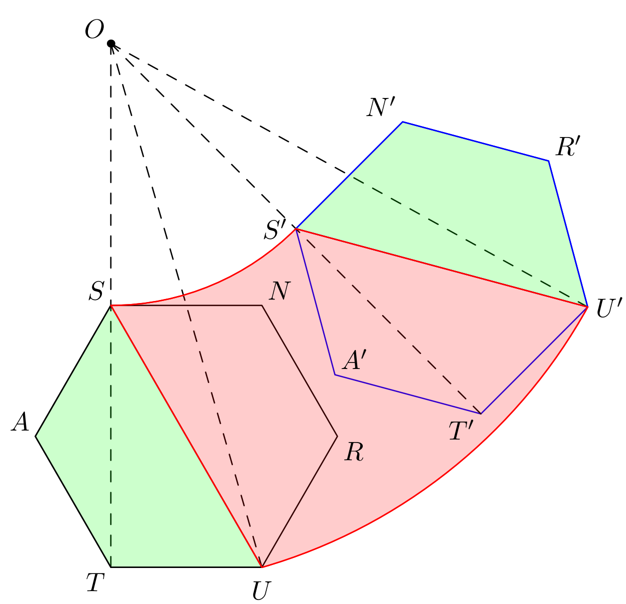
Remark. The alternative answer $495 \cdot 26!$, which can be obtained if a contestant took into account an ordering of deletion of the unused 26 digits, was also accepted.

6. Regular hexagon $SATURN$ (with vertices in counterclockwise order) has side length 2. Point O is the reflection of T over S. Hexagon $SATURN$ is rotated 45° counterclockwise around O. Compute the area its interior traces out during this rotation.

Proposed by: Derek Liu

Answer: $\boxed{5\pi + 6\sqrt{3}}$

Solution:



Let $S'A'T'U'R'N'$ be the rotated hexagon. Note that S is the point on the hexagon closest to O, and U is the farthest. Thus, the region traced between SU and $S'U'$ (shaded in red above) is $1/8$ of an annulus with inner radius OS and outer radius OU. Note that $OS^2 = (2\sqrt{3})^2 = 12$ and

$$OU^2 = OT^2 + TU^2 = (4\sqrt{3})^2 + 2^2 = 52,$$

so this region has area $(52 - 12)\pi/8 = 5\pi$.

The remaining traced area consists of half of the hexagon on each side; specifically, it consists of trapezoids $SATU$ and $U'R'N'S'$ (the regions in green). The hexagon has area $6\sqrt{3}$, so the answer is $\boxed{5\pi + 6\sqrt{3}}$.

7. Io, Europa, and Ganymede are three of Jupiter's moons. In one Jupiter month, they complete exactly I, E, and G orbits around Jupiter, respectively, for some positive integers I, E, and G. Each moon appears as a full moon precisely at the start of each of its orbits. Suppose that in every Jupiter month, there are

- exactly 54 moments of time with at least one full moon,
- exactly 11 moments of time with at least two full moons, and
- at least 1 moment of time with all three full moons.

Compute $I \cdot E \cdot G$.

Proposed by: Derek Liu, Eric Wang

Answer: $\boxed{7350}$

Solution: Note that if two moons appear as full moons exactly m and n times each month, respectively, then they are simultaneously full moons exactly $\gcd(m, n)$ times each month. Using inclusion-exclusion, we can interpret the problem conditions as

$$I + E + G - \gcd(I, E) - \gcd(E, G) - \gcd(I, G) + \gcd(I, E, G) = 54,$$
$$\gcd(I, E) + \gcd(E, G) + \gcd(I, G) - 2\gcd(I, E, G) = 11.$$

From this, we see that $\gcd(I, E, G)$ divides both 54 and 11, so $\gcd(I, E, G) = 1$. Thus,

$$\gcd(I, E) + \gcd(E, G) + \gcd(I, G) = 11 + 2 = 13$$

and

$$I + E + G = 54 + 13 - 1 = 66.$$

Now, consider the values of $\gcd(I, E)$, $\gcd(E, G)$, and $\gcd(I, G)$. They must add to 13, and the gcd of any two of them is $\gcd(I, E, G) = 1$. In particular, since their sum is 13, either 0 or 2 of them are even, so none of them are even. It follows that the only possibilities for the three gcds are $(1, 1, 11)$ and $(1, 5, 7)$. Note that if 11 divides two of I, E, and G, it must divide the third, since it divides their sum $I + E + G = 66$. Hence, $(1, 1, 11)$ is not possible, leaving $(1, 5, 7)$ as the only possibility.

Thus, I, E, and G must be of the form $35a$, $5b$, and $7c$, in some order, for some positive integers a, b, and c. The only solution satisfying $I + E + G = 66$ is $(a, b, c) = (1, 2, 3)$, resulting in I, E and G being 35, 10, and 21, in some order. It can be checked that these values satisfy the given conditions, so $I \cdot G \cdot E = 35 \cdot 10 \cdot 21 = \boxed{7350}$.

8. Let $MARS$ be a trapezoid with $\overline{MA}$ parallel to $\overline{RS}$ and side lengths $MA = 11$, $AR = 17$, $RS = 22$, and $SM = 16$. Point X lies on side $\overline{MA}$ such that the common chord of the circumcircles of triangles MXS and AXR bisects segment $\overline{RS}$. Compute MX.

Proposed by: Pitchayut Saengrungkongka

Answer: $\boxed{\frac{17}{2} = 8.5}$

Solution 1:

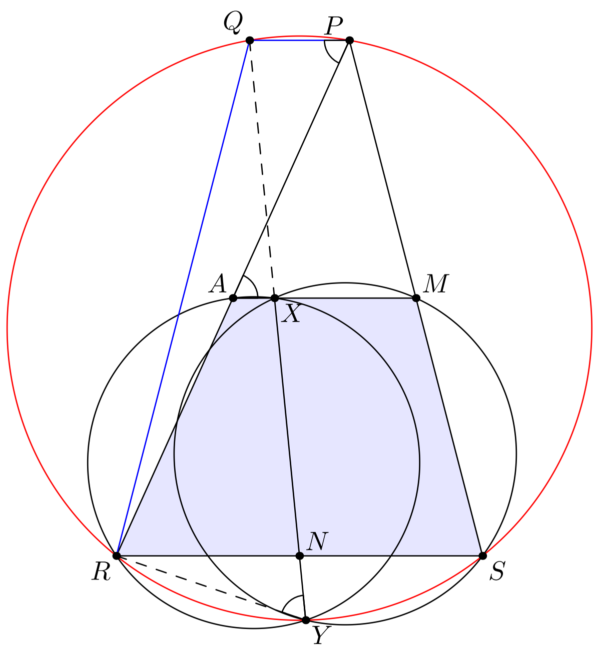
Let Y be the second intersection of the two circumcircles. Let $N = XY \cap SR$, $P = MS \cap AR$, and construct Q on the line through P parallel to SR so that $PQRS$ is an isosceles trapezoid, which is cyclic. Then we have that Y lies on this circle because

$$\begin{aligned}\angle SYR &= \angle SYX + \angle XYR \\ &= \angle PMX + \angle PAX \\ &= 180^\circ - \angle SPR,\end{aligned}$$

so $PSYRQ$ is cyclic. We also have

$$\angle QYR = \angle QPR = \angle PAX = \angle XYR$$

so Q, X, and Y are collinear.



Note that since $\triangle PAM \sim \triangle PRS$ and $RS = 2AM$, we have $QR = PS = 2MS = 32$ and $QS = PR = 2AR = 34$. By Ptolemy's on $PQRS$, we obtain

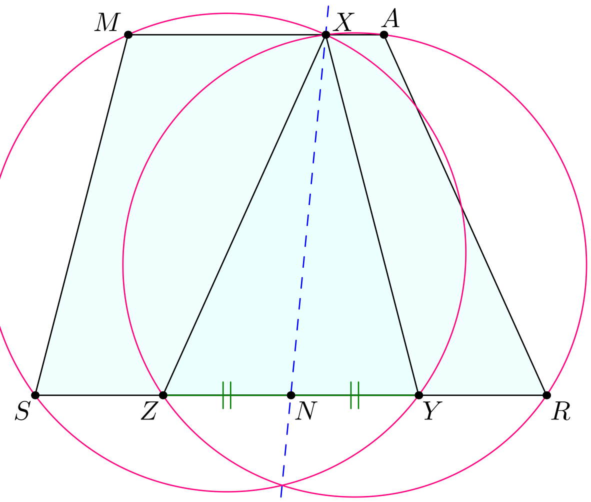
$$PQ = \frac{PR \cdot QS - PS \cdot QR}{RS} = \frac{34^2 - 32^2}{22} = 6.$$

Finally,

$$MX = \frac{PQ + SN}{2} = \frac{6 + 11}{2} = \boxed{\frac{17}{2}}.$$

Solution 2:

Let the circumcircles of $\triangle MXS$ and $\triangle AXR$ intersect line SR again at Y and Z, respectively. Let N be the midpoint of SR. Because N lies on the radical axis of the two circles, we have $SN \cdot NY = RN \cdot NZ$, so $YN = ZN$. As a result, $SY = RZ$.



Let F_M and F_A be the feet of altitudes from M and A to SR, respectively. By Pythagoras' theorem,

$$\sqrt{16^2 - SF_M^2} = \sqrt{17^2 - RF_A^2} \implies RF_A^2 - SF_M^2 = 17^2 - 16^2 = 33.$$

Since $SF_M + RF_A = SR - MA = 11$, we get that $RF_A - SF_M = 33/11 = 3$, so $SF_M = 4$ and $RF_A = 7$. Because quadrilaterals $MXSY$ and $AXZR$ are both isosceles trapezoids, we have $SY = MX + 2 \cdot SF_M = MX + 8$ and $RZ = AX + 2 \cdot RF_A = AX + 14$. Therefore, $MX + 8 = AX + 14$. Since $MX + AX = MA = 11$, we obtain that $MX = \boxed{8.5}$.

9. Triton performs an ancient Neptunian ritual consisting of drawing red, green, and blue marbles from a bag. Initially, Triton has 3 marbles of each color, and the bag contains an additional 3 marbles of each color. Every turn, Triton picks one marble to put into the bag, then draws one marble uniformly at random from the bag (possibly the one he just discarded). The ritual is completed once Triton has 6 marbles of one color and 3 of another. Compute the expected number of turns the ritual will take, given that Triton plays optimally to minimize this value.

Proposed by: Derek Liu

Answer: $\boxed{\frac{91}{6}}$

Solution: Intuitively, an optimal strategy for Triton is for him put a marble of the least common color among his marbles into the bag every turn (with ties broken arbitrarily). This is because to complete the ritual, he must get rid of all the marbles of some color, then minimize the number of marbles of another color. The fastest way to do this is to remove a marble of the color that he wants to minimize at each turn, which would be the least common color. For completeness, we give a rigorous proof of optimality at the end of the solution.

Thus, it is optimal to initially fix a color, then repeatedly put a marble of that color into the bag until Triton doesn't have any marbles of that color left. Once this happens, he should then repeatedly discard the less common of the two remaining colors he has until the ritual is complete.

Without loss of generality, suppose Triton chooses to get rid of his 3 red marbles first. After putting a red marble into the bag on his first turn, 4 of the 10 marbles in the bag will be red. When he draws a marble back from the bag, there is a $\frac{4}{10}$ chance the marble is red (which means his turn did nothing), and a $\frac{6}{10}$ chance the marble is not red (which means Triton now has 2 red marbles). The expected number of turns before the latter scenario happens is $\frac{10}{6}$.

Similarly, when Triton has 2 red marbles and returns one to the bag, the bag will have 5 red and 5 non-red marbles. We therefore expect it to take $\frac{10}{5}$ turns before he has only 1 red marble remaining. Likewise, getting rid of the last red marble will take an expected $\frac{10}{4}$ turns.

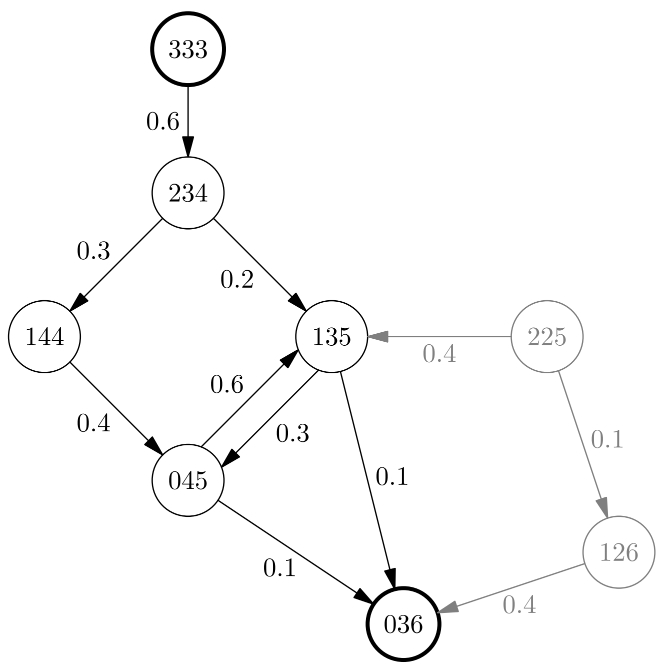
By symmetry, the 3 marbles that replaced Triton's red marbles are equally likely to be any 3 of the 6 non-red marbles that were initially in the bag. There are $\binom{6}{3} = 20$ possible cases and two scenarios to consider:

1. The 3 new marbles are all the same color. There are 2 ways for this to happen. Here, Triton is immediately done with the ritual.
2. The 3 new marbles are not all the same color. There are $20 - 2 = 18$ ways for this to happen. Triton's marble colors will be either 4 green and 5 blue, or 5 green and 4 blue. Without loss of generality, we assume the former. Next turn, Triton will return a green marble to the bag, and he will repeatedly discard the marble he newly draws until he draws the sixth blue marble, at which point the ritual is complete. Only 1 out of the 10 marbles in the bag will be blue when Triton draws, meaning it will take him an expected 10 turns to get the sixth blue marble and complete the ritual.

The total expected number of turns the ritual will take is therefore

$$\frac{10}{4} + \frac{10}{5} + \frac{10}{6} + \left(\frac{2}{20} \cdot 0 + \frac{18}{20} \cdot 10\right) = \boxed{\frac{91}{6}}.$$

As an addendum, we now rigorously prove why the described strategy above is optimal.



The claimed optimal strategy leads to the transition graph shown, where abc denotes a state in which the number of marbles of each color are a, b, and c in some order (it does not matter which). Each state returns to the same state with the remaining probability (for example, state 234 returns to itself with probability $1 - 0.3 - 0.2 = 0.5$.) In other words, the claimed expected number of turns required to complete the ritual from a given state are as follows:

036	135	045	144	234	333	126	225
0	10	10	25/2	27/2	91/6	5/2	21/2

We can prove optimality for this problem as follows:

- Suppose it is possible to improve upon any of the numbers in the above table using a certain strategy. Look at the entry which we can decrease the most, say we can replace x_i with $x_i' = x_i - \Delta$. We can simplify any mixed strategy at this state to some deterministic strategy without increasing its expected remoteness, since we know $x_i - \Delta$ is a weighted average of the values obtained by the possible deterministic strategies.
- If this same strategy is applied on the weights *given in the above table*, then we claim it must also reduce x_i. Suppose it leads to x_j with probability w_{ij}; then $x_i - \Delta = 1 + \sum w_{ij} x_j' \geq 1 + \sum w_{ij}(x_j - \Delta) = (1 + \sum w_{ij} x_j) - \Delta$, where equality can only hold if all x_j for which $w_{ij} \neq 0$ satisfy $x_j' = x_j - \Delta$. We can then apply the same argument to every such x_j, which eventually leads to a contradiction because every node in this graph has a path of nonzero edges leading to 036.
- Thus, $x_i < 1 + \sum w_{ij} x_j$, which reduces verifying optimality to a finite case bash. It is straightforward to consider, for each of the 8 states, the non-optimal choices of marble to remove and then verify that this inequality does not hold.

The general solution can be described as linear programming duality.

10. The orbits of Pluto and Charon are given by the ellipses

$$x^2 + xy + y^2 = 20 \quad \text{and} \quad 2x^2 - xy + y^2 = 25,$$

respectively. These orbits intersect at four points that form a parallelogram. Compute the largest of the slopes of the four sides of this parallelogram.

Proposed by: Pitchayut Saengrungkongka

Answer: $\boxed{\frac{\sqrt{7}+1}{2}}$

Solution 1: For any real number t, adding t times the first orbit equation and $1 - t$ times the second orbit equation shows that all four intersection points must satisfy the equation

$$\Big(t + 2(1-t)\Big)x^2 + \Big(t - (1-t)\Big)xy + \Big(t + (1-t)\Big)y^2 = 20t + 25(1-t)$$

which simplifies to

$$(2-t)x^2 + (2t-1)xy + y^2 = 25 - 5t.$$

We want to choose t so that the left hand side is a perfect square. This choice can be found by equating its discriminant to 0:

$$(2t-1)^2 - 4(2-t) = 0.$$

This simplifies to $4t^2 - 7 = 0$, so $t = \pm\sqrt{7}/2$. Fixing a choice of sign for t, we see all four intersection points must satisfy the equation

$$\left(2 \mp \frac{\sqrt{7}}{2}\right) x^2 + (\pm\sqrt{7} - 1)xy + y^2 = (\text{some constant}),$$

which simplifies to

$$\left(y + \frac{-1 \pm \sqrt{7}}{2} x\right)^2 = (\text{some constant}).$$

The graph of this equation consists of two parallel lines of slope $\frac{1 \pm \sqrt{7}}{2}$, so these lines must be opposite sides of the parallelogram. Hence, the maximum slope is $\boxed{\frac{1+\sqrt{7}}{2}}$.

Solution 2: Let (x_1, y_1) and (x_2, y_2) be two distinct intersection points which are not reflections across the origin; the other two are $(-x_1, -y_1)$ and $(-x_2, -y_2)$. Thus, the two slopes are $m_1 = \frac{y_1 - y_2}{x_1 - x_2}$ and $m_2 = \frac{y_1 + y_2}{x_1 + x_2}$. Since subtracting the two orbit equations yields $x^2 - 2xy = 5$,

$$m_1 + m_2 = \frac{2x_1y_1 - 2x_2y_2}{x_1^2 - x_2^2} = \frac{(x_1^2 - 5) - (x_2^2 - 5)}{x_1^2 - x_2^2} = 1.$$

Likewise, adding the two equations yields $3x^2 + 2y^2 = 45$, so

$$m_1m_2 = \frac{y_1^2 - y_2^2}{x_1^2 - x_2^2} = \frac{1}{2} \cdot \frac{(45 - 3x_1^2) - (45 - 3x_2^2)}{x_1^2 - x_2^2} = -\frac{3}{2}.$$

Thus, the two slopes are roots of $m^2 - m - 3/2 = 0$, and the larger root of this polynomial is $\boxed{\frac{1+\sqrt{7}}{2}}$.