HMMT February 2026

February 14, 2026

Algebra and Number Theory Round

1. A line intersects the graph of $y = x^2 + \frac{2}{x}$ at three distinct points. Given that the x-coordinates of two of the points are 6 and 7, respectively, compute the x-coordinate of the third point.

2. Compute the second smallest positive integer n such that

   - n is divisible by 101, and
   - the decimal representation of n contains the number 2026 as a consecutive sequence of digits.

3. Compute the sum of all positive integers n such that n has at least 6 positive integer divisors and the 6th largest divisor of n is 6.

4. Let a, b, and c be pairwise distinct complex numbers such that

$$\begin{aligned} a^2 + ab + b^2 &= 3(a+b), \\ a^2 + ac + c^2 &= 3(a+c), \\ b^2 + bc + c^2 &= 5(b+c) + 1. \end{aligned}$$

   Compute a.

5. Compute the largest positive integer n such that

$$n \text{ divides } (\lfloor \sqrt{n} \rfloor)!^{n!} + 450.$$

6. The numbers 1, 2, …, 2100 are written on a board. Every second, Mark takes two numbers on the board, a and b, erases them, and replaces them with $\gcd(a,b)$ and $\operatorname{lcm}(a,b)$. Mark stops once any move he makes will not change the numbers written on the board. Compute the number of divisors of the 2026th smallest positive integer written on the board when he finishes.

7. Positive real numbers x, y, and z satisfy the following equations:

$$\begin{aligned} xyz &= 3, \\ (x-y)(y-z)(z-x) &= 4, \\ (x+y)(y+z)(z+x) &= 40. \end{aligned}$$

   Compute the minimum possible value for x.

8. Let $a_0, a_1, a_2, \ldots$ be the unique sequence of nonnegative integers less than 397 with $a_0 = 1$ and

$$a_{n+1}(a_n + 1)^2 \equiv a_n \pmod{397}$$

   for all nonnegative integers n. Given that $a_{2026} = 9$, compute the remainder when $a_0 + a_1 + \cdots + a_{2026}$ is divided by 397.

9. Compute

$$\sum_{k=1}^{\infty} \left(2^{-\lfloor 101k/1 \rfloor} + 2^{-\lfloor 101k/2 \rfloor} + \cdots + 2^{-\lfloor 101k/100 \rfloor} \right).$$

10. Let

$$S = \sum_{k=0}^{2026} k \binom{2k}{k} 2^k.$$

   Compute the remainder when S is divided by 2027. (Note that 2027 is prime.)