HMMT February 2026

February 14, 2026

Combinatorics Round

1. A math test has 4 questions. The topic of each question is randomly and independently chosen from algebra, combinatorics, geometry, and number theory. Given that the math test has at least one algebra question, at least one combinatorics question, and at least one geometry question, compute the probability that this test has at least one number theory question.

2. Jacopo is rolling a fair 4-sided die with faces labeled 1, 2, 3, and 4. He starts with a score of 0. Every time he rolls a face with label i, he adds i to his score, and then replaces the label of that face with 0. Compute Jacopo's expected score after 4 rolls.

3. The numbers 1, 2, 3, 4, 5, 6, and 7 are written on a blackboard in some order. Jacob repeatedly swaps numbers at adjacent positions on the blackboard until the numbers are sorted in ascending order. Compute the number of initial orderings for which it is possible that the number 4 was included in a swap at most once.

4. Sarunyu has a stick of length 1 with one endpoint marked in red. Every minute, he picks one of his sticks uniformly at random and breaks it into two halves of equal length. Compute the expected length of the stick with the red endpoint after 5 minutes.

5. Let S be the set of positive integer divisors of 10^9. Compute the number of subsets T of S such that
   - for every element s of S, exactly one of s and $10^9/s$ is in T, and
   - for every element t of T, all positive integer divisors of t are in T.

6. Derek currently owes π units of a currency called Money of Indiscrete Type, or MIT for short. Every day, the following happens:
   - He flips a fair coin to decide how much of his debt to pay. If he flips heads, he decreases his debt by 1 MIT. If he flips tails, he decreases his debt by 2 MITs. If Derek's debt ever becomes nonpositive, Derek becomes debt-free.
   - Afterwards, his remaining debt doubles.

   Compute the probability that Derek ever becomes debt-free. (MITs are continuous, so the debt is never rounded.)

7. Let S be the set of vertices of a right prism whose bases are regular decagons $A_1A_2\dots A_{10}$ and $B_1B_2\dots B_{10}$. A plane, not passing through any vertex of S, partitions the vertices of S into two sets, one of which is M. Compute the number of possible sets M that can arise out of such a partition.

8. A regular hexagon with side length 4 is subdivided into a lattice of 96 equilateral triangles of side length 1. Let S be the set of all vertices of this lattice. Compute the number of nondegenerate triangles with vertices in S that contain the center of the hexagon strictly in their interior.

9. Let A_1, A_2, A_3, $\dots$ be a sequence of finite nonempty sets of positive integers. Given that $|A_i \cap A_j| = \gcd(i, j)$ for all (not necessarily distinct) positive integers i and j, compute the minimum possible value of
$$\sum_{d|250} \max A_d,$$
   where the sum ranges over all positive integer divisors d of 250.

   (For a finite nonempty set S, we define $\max S$ as the largest element of S.)

10. Let S be the set of all ordered pairs (x, y) of nonnegative integers $0 \le x \le 19$ and $0 \le y \le 2$. Compute the number of permutations (x_1, y_1), (x_2, y_2), $\dots$, (x_{60}, y_{60}) of the elements of S such that

- $y_1 = 2$ and $y_{60} = 0$;
- for all nonnegative integers $1 \le i \le 59$, exactly one of the following holds:
  - $x_i = x_{i+1}$ and $|y_i - y_{i+1}| = 1$,
  - $y_i = y_{i+1}$ and $x_i - x_{i+1}$ is -1 or 19.