HMMT February 2026

February 14, 2026

Combinatorics Round

1. A math test has 4 questions. The topic of each question is randomly and independently chosen from algebra, combinatorics, geometry, and number theory. Given that the math test has at least one algebra question, at least one combinatorics question, and at least one geometry question, compute the probability that this test has at least one number theory question.

   Proposed by: Jacob Paltrowitz

   Answer: $\boxed{\frac{2}{5} = 0.4}$

   Solution: The given condition fixes the topic for three of the four questions on the test. We consider two cases for the topic of the remaining question:

   - If the remaining question is algebra, combinatorics, or geometry, there are $\frac{4!}{2!} = 12$ possible orderings of question topics.
   - If the remaining question is number theory, there are $4! = 24$ possible orderings of question topics.

   The desired conditional probability is found by taking the number of above orderings with at least one number theory over the total number of orderings, which is

$$\frac{24}{24 + 12 + 12 + 12} = \boxed{\frac{2}{5}}.$$

2. Jacopo is rolling a fair 4-sided die with faces labeled 1, 2, 3, and 4. He starts with a score of 0. Every time he rolls a face with label i, he adds i to his score, and then replaces the label of that face with 0. Compute Jacopo's expected score after 4 rolls.

   Proposed by: Srinivas Arun

   Answer: $\boxed{10(1 - (\frac{3}{4})^4) = \frac{875}{128}}$

   Solution: Let S_i be the total score contribution of the face initially labeled i. If this face is rolled at least once, then $S_i = i$. Otherwise, $S_i = 0$. The probability that this face is never rolled is $(\frac{3}{4})^4$. Therefore,

$$\mathbb{E}[S_i] = i\left(1 - \left(\frac{3}{4}\right)^4\right).$$

   By linearity of expectation, Jacopo's expected score is

$$\begin{aligned}\mathbb{E}[S_1 + S_2 + S_3 + S_4] &= \mathbb{E}[S_1] + \mathbb{E}[S_2] + \mathbb{E}[S_3] + \mathbb{E}[S_4] \\ &= (1 + 2 + 3 + 4)\left(1 - \left(\frac{3}{4}\right)^4\right) \\ &= \boxed{\frac{875}{128}}\end{aligned}$$

3. The numbers 1, 2, 3, 4, 5, 6, and 7 are written on a blackboard in some order. Jacob repeatedly swaps numbers at adjacent positions on the blackboard until the numbers are sorted in ascending order.

Compute the number of initial orderings for which it is possible that the number 4 was included in a swap at most once.

Proposed by: Sebastian Attlan

Answer: $\boxed{324}$

Solution: We split into cases based on whether 4 is part of 0 or 1 swap.

- **If** 4 **is part of no swaps**, then 1, 2, and 3 all start on the left of 4, while 5, 6, and 7 all start on the right of 4. Regardless of the initial ordering of the 1, 2, and 3, it is possible to rearrange them into the correct order without performing swaps involving 4. Similarly, the initial ordering of the 4, 5, and 6 is unconstrained. There are 3! ways to order the numbers on each side of 4, so the number of such permutations is $3!^2 = 36$.
- **If** 4 **is part of one swap**, there are 6 ways to pick the number it needs to swap with. Regardless of which number is selected, this fixes four numbers on one side of 4 and two numbers on the other. By an analogous argument to the first case, the ordering within each side is unconstrained. Thus, the number of such permutations is $6 \cdot 4! \cdot 2! = 288$.

Thus, the answer is $36 + 288 = \boxed{324}$.

4. Sarunyu has a stick of length 1 with one endpoint marked in red. Every minute, he picks one of his sticks uniformly at random and breaks it into two halves of equal length. Compute the expected length of the stick with the red endpoint after 5 minutes.

Proposed by: Derek Liu

Answer: $\boxed{\frac{\binom{10}{5}}{2^{10}} = \frac{63}{256}}$

Solution: Let E_n denote the expected length of the stick with the red endpoint after n minutes.

Every minute, Sarunyu breaks one stick into two, so right before the nth minute, Sarunyu has n sticks. The probability that he selects the stick with the red endpoint is $\frac{1}{n}$. Therefore, if the stick with the red endpoint had length x before the nth minute, then it will have length $\frac{x}{2}$ after the nth minute with probability $\frac{1}{n}$, and length x after the n^{th} minute with probability $\frac{n-1}{n}$. Hence, the expected length of the stick after the n^{th} minute is $(\frac{1}{n} \cdot \frac{1}{2} + \frac{n-1}{n})x = \frac{2n-1}{2n}x$, regardless of the value of x. By linearity of expectation, this means

$$E_n = \frac{2n-1}{2n} E_{n-1}.$$

We conclude that the expected length after 5 minutes is

$$E_5 = \left(\frac{1}{2}\right)\left(\frac{3}{4}\right)\cdots\left(\frac{9}{10}\right) = \boxed{\frac{63}{256}}.$$

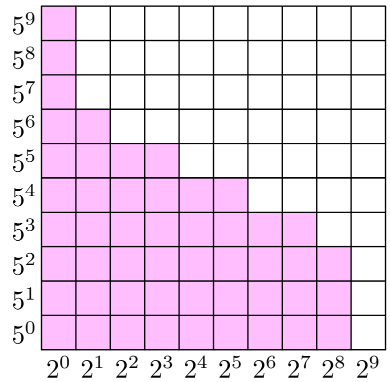
5. Let S be the set of positive integer divisors of 10^9. Compute the number of subsets T of S such that

- for every element s of S, exactly one of s and $10^9/s$ is in T, and
- for every element t of T, all positive integer divisors of t are in T.

Proposed by: Jackson Dryg

Answer: $\boxed{252}$

Solution: Represent the elements of S as cells in a 10×10 grid, where each cell corresponds to the product of the number left of its row and the number below its column. The shaded squares represent one possible value for T.



Number the columns $0, 1, \ldots, 9$ from left to right.

The second condition implies that for every shaded cell, all cells to the left and/or below that cell are also shaded. This means that the shaded cells in a column must be the lowest k cells in that column, for some k. Let the *height* of a column be the number of shaded cells in it. Then the column heights must be nonincreasing from left to right.

The first condition implies that for all $0 \leq i \leq 9$, column i and column $9 - i$ have heights that add up to 10. This means that if we choose the heights of columns 0 through 4, the heights of the other columns are uniquely determined. By the second condition, the height of column 4 must be at least the height of column 5. By the first condition, the heights of these two columns add to 10, so the height of column 4 is at least 5. Let the heights of the first five columns be a, b, c, d, e from left to right; then we know that $10 \geq a \geq b \geq c \geq d \geq e \geq 5$.

Any (a, b, c, d, e) satisfying this inequality gives rise to exactly one valid subset T. Hence the answer is the number of such (a, b, c, d, e). This is the same as the number of 5-tuples $(a+4, b+3, c+2, d+1, e)$ with $14 \geq a + 4 > b + 3 > c + 2 > d + 1 > e \geq 5$, which is $\binom{10}{5} = 252$.

6. Derek currently owes π units of a currency called Money of Indiscrete Type, or MIT for short. Every day, the following happens:

- He flips a fair coin to decide how much of his debt to pay. If he flips heads, he decreases his debt by 1 MIT. If he flips tails, he decreases his debt by 2 MITs. If Derek's debt ever becomes nonpositive, Derek becomes debt-free.
- Afterwards, his remaining debt doubles.

Compute the probability that Derek ever becomes debt-free. (MITs are continuous, so the debt is never rounded.)

Proposed by: Derek Liu

Answer: $\boxed{\frac{4-\pi}{2} = 2 - \frac{\pi}{2}}$

Solution: Instead of the debt doubling, we can imagine that every day, Derek's payment halves. Label the first day as 0; then, on day i, he randomly decides between paying off 2^{-i} and $2 \cdot 2^{-i}$ of his remaining debt (which never increases). It is clear that this formulation of the problem is equivalent.

Suppose that on day i, Derek pays $(1 + b_i) \cdot 2^{-i}$ of his debt, so each b_i is either 0 or 1 with equal probability. Then, he pays off his full debt if and only if

$$\sum_{i=0}^{\infty}(1 + b_i)2^{-i} \geq \pi \iff \sum_{i=0}^{\infty} b_i 2^{-i} \geq \pi - 2.$$

Note that $\sum_{i=0}^{\infty} b_i 2^{-i}$ is a uniformly distributed real number from 0 to 2, so the answer is

$$\frac{2-(\pi-2)}{2} = \boxed{\frac{4-\pi}{2}}.$$

7. Let S be the set of vertices of a right prism whose bases are regular decagons $A_1A_2\ldots A_{10}$ and $B_1B_2\ldots B_{10}$. A plane, not passing through any vertex of S, partitions the vertices of S into two sets, one of which is M. Compute the number of possible sets M that can arise out of such a partition.

Proposed by: Tiger Zhang

Answer: $\boxed{1574}$

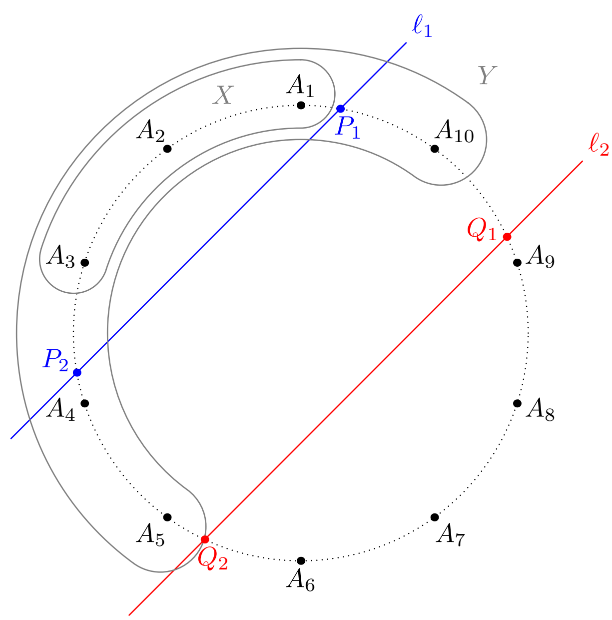
Solution: We first deal with edge cases. Let $A = \{A_1, \ldots, A_{10}\}$ and $B = \{B_1, \ldots, B_{10}\}$ be the two sets of 10 vertices that lie on a single base.

- The empty set, all 20 vertices, A, and B are all possibilities for M.
- If M contains all of A and between 1 and 9 vertices of B (inclusive), then M can contain any consecutive sequences of vertices from B. For each n between 1 and 9, there are 10 such sequences of n vertices, so there are $9 \cdot 10 = 90$ such sets M.
- Likewise, there are 90 possible sets containing all of B and between 1 and 9 vertices of A.
- The complements of the 180 sets from the previous cases are also possibilities for M.

This gives us 364 edge cases. Henceforth, we assume M contains between 1 and 9 vertices on each base.

Let the plane cut $A_1A_2\ldots A_{10}$ at line ℓ_1 and $B_1B_2\ldots B_{10}$ at line ℓ_2. Note that M consists of the vertices of A lying on one side of ℓ_1 (call this set X), along with the vertices of B lying on the corresponding side of ℓ_2 (call this set Y). Moreover, ℓ_1 and ℓ_2 are parallel, and any parallel lines ℓ_1 and ℓ_2 are achievable by some plane.

Therefore, we may make the following simplification to the problem: we project the prism onto a single decagonal base $A_1A_2\ldots A_{10}$, and we count the number of pairs (X, Y) of subsets of vertices of this decagon such that X and Y can be cut by parallel lines ℓ_1 and ℓ_2.



Let Γ be the circumcircle of $A_1A_2\dots A_{10}$. Let ℓ_1 intersect Γ at P_1 and P_2, and let ℓ_2 intersect Γ at Q_1 and Q_2 (with $P_1Q_1 < P_2Q_1$). Then X is precisely the set of vertices of $A_1A_2\dots A_{10}$ lying on arc P_1P_2, and Y is precisely the set of vertices of $A_1A_2\dots A_{10}$ lying on arc Q_1Q_2 (with the two arcs going in the same direction). The key claim is as follows.

Claim 1. The positive difference between the numbers of vertices of A lying on arc P_1Q_1 and lying on arc P_2Q_2 is at most 1.

Proof. Suppose for the sake of contradiction that this is not the case. Without loss of generality, assume k vertices of A lie on arc P_1Q_1, and at least $k+2$ vertices of A lie on arc P_2Q_2. Then arc P_1Q_1 has measure less than $\frac{2\pi}{10}(k+1)$, while arc P_2Q_2 has measure greater than $\frac{2\pi}{10}(k+1)$. But since ℓ_1 and ℓ_2 are parallel, these arcs must have the same measure, contradiction. □

Now we are ready to finish the problem. We will count the number of possible pairs (X, Y) given their sizes $x = |X|$ and $y = |Y|$. We casework by their parities. In both cases, we assume without loss of generality that $x \geq y$ and $X = \{A_1, A_2, \dots, A_x\}$; the same arguments apply if $y \geq x$ or X is rotated.

- If x and y have the same parity, let $x - y = 2k$, the total number of vertices of A on arcs P_1Q_1 and P_2Q_2. Our key claim implies that each arc must have k of these points, so we must have $Y = \{A_{k+1}, \dots, A_{x-k}\}$, which gives 1 possibility.
- If x and y have opposite parity, let $x - y = 2k - 1$, the total number of vertices of A on arcs P_1Q_1 and P_2Q_2. Our key claim implies that one of these arcs must have $k-1$ points and the other must have k, so we must have $Y = \{A_k, \dots, A_{x-k}\}$ or $Y = \{A_{k+1}, \dots, A_{x-k+1}\}$, which gives 2 possibilities.

There are $(5)(5) + (4)(4) = 41$ cases where x and y have the same parity, and $(5)(4) + (4)(5) = 40$ cases where x and y have opposite parities. Accounting for the fact that there are 10 possible sets X for each x, we get a total of $10(41 \cdot 1 + 40 \cdot 2) = 1210$ possibilities.

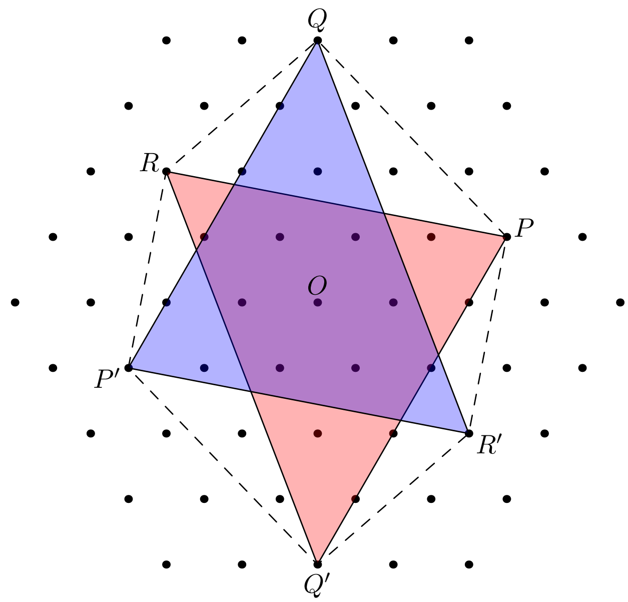
Adding back the edges cases, we get a final answer of $364 + 1210 = \boxed{1574}$.

8. A regular hexagon with side length 4 is subdivided into a lattice of 96 equilateral triangles of side length 1. Let S be the set of all vertices of this lattice. Compute the number of nondegenerate triangles with vertices in S that contain the center of the hexagon strictly in their interior.

   Proposed by: Jessica Wan, Karn Chutinan

   Answer: $\boxed{6992}$

   Solution:



By looking at hexagonal rings of lattice vertices around the center, the cardinality of S is $1+6+12+18+24=61$.

Since triangles with the center O as a vertex cannot be counted, remove O from S to yield a set S^* containing 60 vertices.

Consider any 3 points P, Q, R in S^* such that no two are collinear with O. Let P', Q', R' be their respective reflections about O. Relabel the points such that that P, Q, R lie on the same side of some line passing through O, in that order. Of the triangles with vertices in $\{P, P', Q, Q', R, R'\}$, the only ones that strictly contain O in their interior are $\triangle PQ'R$ and $\triangle P'QR'$.

Thus, the total number of triangles strictly containing O is twice the number of ways to pick the unordered pairs $\{P, P'\}$, $\{Q, Q'\}$, and $\{R, R'\}$ such that no two of P, Q, and R are collinear with O. (Note that it makes no difference what order the pairs are in, nor does it matter which point in the pair is P and which is P', etc.) We call such a set of points *nondegenerate*.

There are $\binom{30}{3}$ ways to select the opposing pairs $\{P, P'\}$, $\{Q, Q'\}$, and $\{R, R'\}$ from the 30 pairs of opposite points up to ordering. It remains to subtract out the number of such selections which are not nondegenerate. We count these in the following ways:

- Two of $\{P, P'\}$, $\{Q, Q'\}$, $\{R, R'\}$ lie on a line connecting the midpoints of opposite edges of the hexagon. There are 3 such lines, each of which has 2 pairs of opposing points; the last pair can then be selected as any of the other $30-2=28$. This gives a count of $3 \cdot \binom{2}{2} \cdot 28 = 84$.
- Exactly two of $\{P, P'\}$, $\{Q, Q'\}$, $\{R, R'\}$ lie on the line connecting opposite vertices of the hexagon. There are 3 such lines, each of which has 4 pairs of opposing points; the last pair can then be selected as any of the other $30-4=26$. This gives a count of $3 \cdot \binom{4}{2} \cdot 26 = 468$.
- All three of $\{P, P'\}$, $\{Q, Q'\}$, $\{R, R'\}$ lie on the line connecting opposite vertices of the hexagon. Like above, this gives a count of $3 \cdot \binom{4}{3} = 12$.

This gives us 564 total degenerate cases. Our answer is therefore

$$2\left(\binom{30}{3} - 564\right) = \boxed{6992}.$$

9. Let A_1, A_2, A_3, ... be a sequence of finite nonempty sets of positive integers. Given that $|A_i \cap A_j| = \gcd(i, j)$ for all (not necessarily distinct) positive integers i and j, compute the minimum possible value of
$$\sum_{d|250} \max A_d,$$
where the sum ranges over all positive integer divisors d of 250.

(For a finite nonempty set S, we define $\max S$ as the largest element of S.)

Proposed by: Jackson Dryg

Answer: $\boxed{499}$

Solution: First we characterize the sets A_i.

Set $i = j$ to obtain $|A_i| = i$ for all i. If $i \mid j$, then $|A_i \cap A_j| = i$, so $A_i \subseteq A_j$. For any i, j, it is true that $A_i \cap A_j = A_{\gcd(i,j)}$. This is because $A_{\gcd(i,j)}$ is a subset of both A_i and A_j, and $|A_{\gcd(i,j)}| = \gcd(i, j)$.

For all n, let B_n be the set of all elements of A_n that do not appear in A_k for any $k \le n$. By definition, all B_i are disjoint.

Claim 1. For all positive integers n, $A_n = \bigcup_{d|n} B_d$.

Proof. Note that $B_d \subseteq A_d \subseteq A_n$ for all $d \mid n$.

By construction of the B_i, any element of A_n must be in some B_k with $k \le n$. It suffices to show that if $k \nmid n$, then $A_n \cap B_k = \emptyset$.

Note that $A_n \cap B_k \subseteq A_n \cap A_k = A_{\gcd(n,k)}$. If $k \nmid n$, $\gcd(n, k) < k$, so no element of $A_{\gcd(n,k)}$ appears in B_k (by definition of B_k). This implies $A_n \cap B_k = \emptyset$, as needed. □

Claim 2. We have $|B_n| = \varphi(n)$ for all positive integers n, where φ is Euler's totient function.

Proof. The first claim gives $n = |A_n| = \sum_{d|n} |B_d|$.

It can be shown that $f(n) = \varphi(n)$ is the unique function f satisfying $\sum_{d|n} f(n) = n$ (by induction, or Mobius inversion). □

Given this characterization of the A_i, we're ready to solve the main problem. The answer is $\boxed{499}$. The construction is $B_1 = \{1\}$, $B_2 = \{2\}$, $B_5 = \{3, 4, 5, 6\}$, $B_{10} = \{7, 8, 9, 10\}$, $B_{25} = \{11, \dots, 30\}$, $B_{50} = \{31, \dots, 50\}$, $B_{125} = \{51, \dots, 150\}$, $B_{250} = \{151, \dots, 250\}$.

For the bound, note that $\sum_{d|n} \max(A_d) \ge \sum_{d|n} \max(B_d)$. These B_i's have 250 elements in total; clearly it is optimal to have them be the numbers $1, 2, \dots, 250$ in some order. We have the following claim.

Claim 3. Let $S_1, S_2, \dots, S_k$ be disjoint sets of positive integers with sizes $n_1 \le n_2 \le \dots \le n_k$ respectively, and let $S = \bigcup_{i=1}^{k} S_i$. Then $\sum_{i=1}^{k} \max(S_k)$ is minimized when S_1 contains the n_1 smallest elements of S, S_2 contains the n_2 next smallest elements of S, etc.

Proof. Induct on k, with the base case $k = 1$ being trivial.

For the inductive step, suppose the claim is true for $k - 1$; we'll show it's true for k. Let the elements of S be $a_1 < a_2 < \dots < a_{n_1+n_2+\dots+n_k}$. Suppose the maximum of S_k is a_m, for $m < n_1 + n_2 + \dots + n_k$. Suppose also that $a_{n_1+n_2+\dots+n_k}$ is the maximum of S_ℓ. Take all elements greater than a_m in S_ℓ, and swap them with elements in S_k. (This is possible because $|S_k| > |S_\ell|$. This swaps the maxima of S_k and S_ℓ, so this doesn't change the sum.)

Now that the maximum of S_k is $a_{n_1+n_2+\dots+n_k}$, swap S_k's elements with elements the other sets so that S_k contains the n_k largest elements in S. These moves don't change the maximum of S_k, and they can only increase the maxima of the other sets. So these moves do not increase the sum $\sum_{i=1}^{k} \max(S_k)$.

Now apply the inductive hypothesis on $S_1, S_2, \dots, S_{k-1}$. □

Because $|B_1| < |B_2| < |B_5| < |B_{10}| < |B_{25}| < |B_{50}| < |B_{125}| < |B_{250}|$, the claim gives that the sum is minimized with the construction given above, so we are done.

10. Let S be the set of all ordered pairs (x, y) of nonnegative integers $0 \le x \le 19$ and $0 \le y \le 2$. Compute the number of permutations (x_1, y_1), (x_2, y_2), ..., (x_{60}, y_{60}) of the elements of S such that

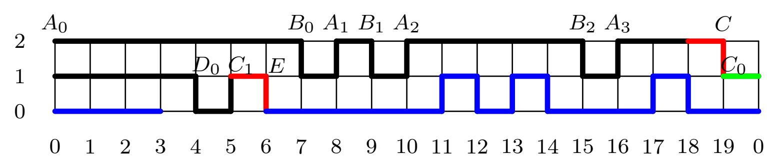
- $y_1 = 2$ and $y_{60} = 0$;
- for all nonnegative integers $1 \le i \le 59$, exactly one of the following holds:
  - $x_i = x_{i+1}$ and $|y_i - y_{i+1}| = 1$,
  - $y_i = y_{i+1}$ and $x_i - x_{i+1}$ is -1 or 19.

Proposed by: Jackson Dryg

Answer: $\boxed{20460}$

Solution: Without loss of generality, Mark starts at $(0, 2)$; we'll multiply by 20 at the end.

Here is an example path:



Note that the two red segments adjacent to point C must be part of any path. Let $A_0, B_0, A_1, B_1 \ldots, A_n, B_n = C$ denote the corners of the path that lie on the line $y = 2$. The B_i are points where the path goes down from the line $y = 2$, and the A_i are points where it comes back up. Terminology:

- Let the entire path between A_0 and C be a *ridge* (abbreviated as ridge A_0C), and let the *length* of the ridge be the length A_0C.
- Let the *head* of the ridge be segment A_0B_0. (The head may have length 0, if the path goes down immediately from point A_0.)
- Let a *valley* be the segment of the path between B_i and A_{i+1}. Let the *depth* of a valley be the number of units below line B_iA_{i+1} the valley reaches (so depth is either 1 or 2). Let the *length* of a valley be the length B_iA_{i+1}.
- Let a *hill* be segment A_iB_i for $i > 0$.

Because of the position of ridge A_0C, the two red segments adjacent to station E must also be part of the path. Consider the section of the path between C_0 and E; it lies on or below the line $y = 1$. Let $C_0, D_0, C_1, D_1, \ldots, C_n, D_n = E$ denote the corners of the path that lie on the line $y = 1$. We can also consider this section of the path from C_0 to E a ridge, with the same terminology.

The following claims apply to both ridges A_0C and C_0E.

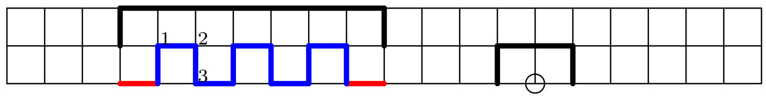
Claim 1. All valleys have width 1.

Proof. Suppose not; then any vertex on the line $y = 2$ between B_i and A_{i+1} (or D_i and C_{i+1}) is isolated from the path, contradiction. □

Claim 2. Hills that are part of ridge A_0C have odd length; hills that are part of ridge C_0E have length 1.

Proof. For the first part, see the diagram below. Consider any hill of A_0C (in black). The red segments are forced to be part of the path. By considering vertices labeled 1, 2, 3, ..., in order, the path between the red segments is uniquely determined, and is only possible when the hill has odd length.

For the second part, see the diagram on the right. If the hill has length greater than 1, then there is an isolated vertex.



□

Claim 3. The head of a ridge has odd length.

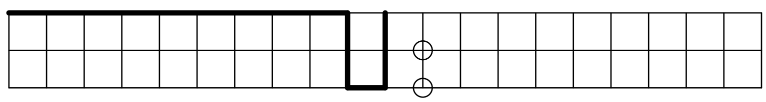
Proof. Note that ridge A_0C has length 19. Because all valleys and hills have odd length, all segments B_iB_{i+1} for $i \geq 0$ have even length. Since $A_0C = 19 = A_0B_0 + B_0B_1 + B_1B_2 + \cdots + B_{n-1}C$, A_0B_0 must have odd length.

Next, note that C_0E has the same length as A_0B_0, the head of A_0C, so it has odd length. By a similar argument, all D_iD_{i+1} have even length, so C_0D_0 has odd length. □

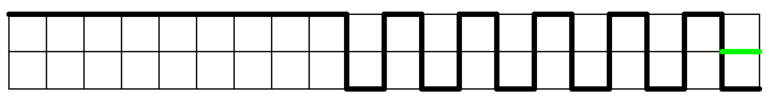
This claim means that the green segment attached to C_0 must be part of any path, since the head C_0D_0 of ridge C_0E has length at least 1.

Claim 4. All valleys have depth 1.

Proof. Suppose a valley had depth 2. Consider the two circled vertices:



These two circled vertices force the next hill to have width 1, and the next valley to have depth 2. This forces the next hill to have width 1 and the next valley to have depth 2, and so on. This continues until C_0. (All hills and valleys have odd length; this ensures the sequence of valleys hits C_0 in this way.) But then the path doesn't contain the green segment:



This is a contradiction. □

Claim 5. Once ridges A_0C and C_0E are chosen, the rest of the path is uniquely determined.

Proof. This is clear from the picture at the beginning; the rest of the path is shown in blue. □

Now we can start counting. First consider the possibilities for ridge A_0C. Each choice of the lengths $(A_0B_0, B_0B_1, \ldots, B_{n-1}C)$ corresponds to exactly one possible ridge. Since all valleys have width 1 and all hills have odd length, each length B_iB_{i+1} must be even.

Suppose the head A_0B_0 has length $19 - 2k$. Then the number of ways to fill in the remaining lengths is the number of partitions of $2k$ into even numbers, which is 1 if $k = 0$ and 2^{k-1} if $k \geq 1$.

Next, given a ridge A_0C, we must choose a ridge C_0E. Note that C_0E has the same length as A_0B_0, so $19 - 2k$. Each choice of the lengths $(C_0D_0, D_0D_1, \ldots, D_{n-1}E)$ corresponds to exactly one possible ridge. Since all hills and valleys have length 1, each D_iD_{i+1} is 2. The possibilities are $(19 - 2k), (17 - 2k, 2), (15 - 2k, 2, 2), \ldots, (1, 2, 2, \ldots, 2)$; there are $10 - k$ possibilities.

So the number of ways to choose both A_0C and C_0E, where the head of A_0C has length $19-2k$, is 10 if $k=0$ and $(10-k)2^{k-1}$ if $k \geq 1$. Summing over all possible head lengths,

$$\begin{aligned}
10 + \sum_{k=1}^{9}(10-k)2^{k-1} &= 10 + \sum_{k=0}^{8}(9-k)2^k \\
&= 10 + \sum_{k=0}^{8}\sum_{\ell=0}^{8-k} 2^k \\
&= 10 + \sum_{\ell=0}^{8}\sum_{k=0}^{8-\ell} 2^k \\
&= 10 + \sum_{\ell=0}^{8} 2^{9-\ell} - 1 \\
&= 10 - 9 + \sum_{\ell=1}^{9} 2^\ell \\
&= 1 + 2^{10} - 2 = 2^{10} - 1 = 1023.
\end{aligned}$$

The final answer is $20 \cdot 1023 = \boxed{20460}$.