HMMT February 2026

February 14, 2026

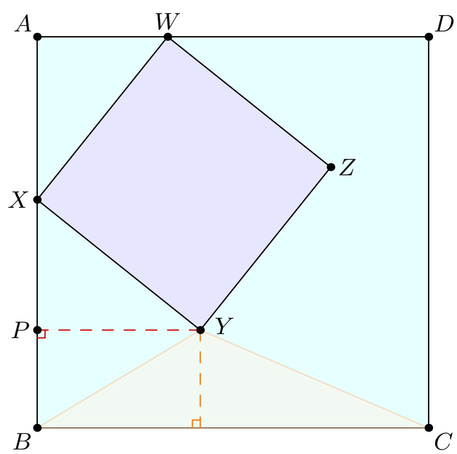
Geometry Round

1. Let $ABCD$ and $WXYZ$ be squares such that W lies on segment $\overline{AD}$, X lies on segment $\overline{AB}$, and points Y and Z lie strictly inside $ABCD$. Given that $AW = 4$, $AX = 5$, and $AB = 12$, compute the area of triangle $\triangle BCY$.

   Proposed by: Pitchayut Saengrungkongka

   Answer: $\boxed{18}$

   Solution:



Let P be the foot from Y to AB. Notice that

- $\angle AWX = 90^\circ - \angle AXY = \angle PXY$
- $\angle WAX = \angle XPY = 90^\circ$
- $WX = XY$.

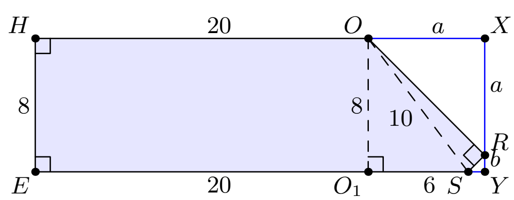
Hence, triangles WAX and XPY are congruent, so $XP = AW = 4$ and $BP = AB - AX - XP = 12 - 5 - 4 = 3$. Thus, the altitude from Y to BC has length 3, implying that the area of BCY is $\frac{1}{2} \cdot 12 \cdot 3 = \boxed{18}$.

2. Let $HORSE$ be a convex pentagon such that $\angle EHO = \angle ORS = \angle SEH = 90^\circ$ and $\angle HOR = \angle RSE = 135^\circ$. Given that $HO = 20$, $SE = 26$, and $OS = 10$, compute the area of $HORSE$.

   Proposed by: Jason Mao

   Answer: $\boxed{191}$

   Solution:



Let O_1 be the foot of the perpendicular from O onto SE. We have $O_1S = 6$, so $HE = OO_1 = 8$. Hence, the area of trapezoid $HOSE$ is $\frac{(20+26)\cdot 8}{2} = 184$.

Now, draw circumscribing rectangle $HXYE$ such that R lies on XY. Note that $\triangle OXR$ and $\triangle RYS$ are both isosceles right triangles. Let $OX = XR = a$ and $RY = YS = b$. We have

$$a - b = O_1S = 6$$

and

$$a + b = XY = HE = 8.$$

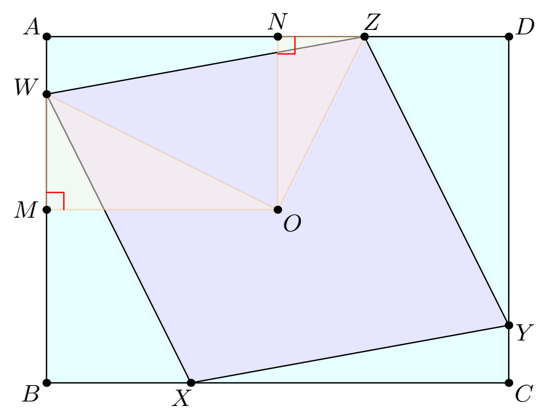
Thus, we have $a = 7$ and $b = 1$. Hence, $OR = 7\sqrt{2}$ and $RS = \sqrt{2}$, giving $[ORS] = 7$. Finally, $[HORSE] = [HOSE] + [ORS] = 184 + 7 = \boxed{191}$.

3. Let $ABCD$ be a rectangle with $AB = 12$ and $BC = 16$. Points W, X, Y, and Z lie on sides $\overline{AB}$, $\overline{BC}$, $\overline{CD}$, and $\overline{DA}$, respectively, such that $WXYZ$ is a rhombus with area 120. Compute XY.

   Proposed by: Pitchayut Saengrungkongka

   Answer: $\boxed{5\sqrt{5}}$

   Solution:



Let O be the intersection of the diagonals WY and XZ. Since $OW = OY$, we see that O is equidistant from lines AB and CD. Similarly, O is equidistant from lines BC and DA. Thus, O is the center of the rectangle $ABCD$ too.

Let M and N be the midpoint of AB and AD. Since WY is perpendicular to XZ, we get that $\triangle OMW \sim \triangle ONZ$. Therefore, $OW : OZ = OM : ON = 8 : 6 = 4 : 3$. Let $OW = 4x$ and $OZ = 3x$. Then the diagonals of the rhombus have length $8x$ and $6x$. Since the rhombus have area 120, we get that

$$\frac{1}{2}(8x)(6x) = 120 \implies x = \sqrt{5}.$$

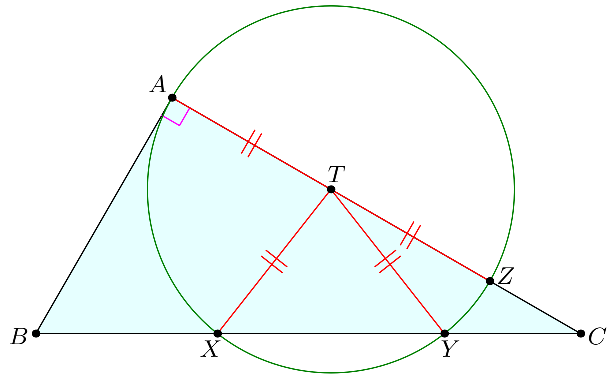
Finally, by Pythagorean theorem, the side length of rhombus is $\sqrt{(3x)^2 + (4x)^2} = 5x = \boxed{5\sqrt{5}}$.

4. Let ABC be a triangle with $\angle BAC = 90^\circ$. Points X and Y are such that B, X, Y, and C lie on segment $\overline{BC}$ in that order, $BX = 4$, $XY = 5$, and $YC = 3$. Let T be a point lying on segment $\overline{AC}$ such that $TA = TX = TY = \ell$ for some ℓ. Compute ℓ.

   Proposed by: Jason Mao

   Answer: $\boxed{\frac{7}{3}\sqrt{3}}$

   Solution:



Draw circle ω with center T and radius ℓ, which passes through points A, X, Y. Since ω is tangent to BA at A, taking the power of B with respect to ω yields

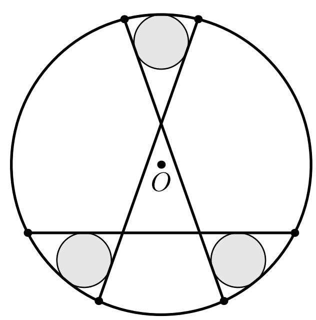
$$BA^2 = BX \cdot BY = 4 \cdot 9 \implies BA = 6.$$

Thus we get $AC = \sqrt{BC^2 - BA^2} = 6\sqrt{3}$. Let Z be the reflection of A across T, which is on ω. Taking the power of C with respect to ω yields

$$CY \cdot CX = CZ \cdot CA \implies 3 \cdot 8 = (6\sqrt{3} - 2\ell) \cdot 6\sqrt{3}$$

Solving this equation, we get that $\ell = \boxed{\frac{7\sqrt{3}}{3}}$.

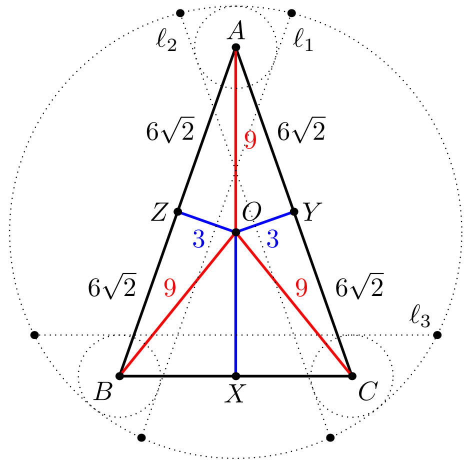
5. In the diagram below, three circles of radius 2 are internally tangent to a circle Ω centered at O of radius 11, and three chords of Ω are each tangent to two of the three circles. Given that O lies inside the triangle formed by the three chords and two of the chords have length $4\sqrt{30}$, compute the length of the third chord.



Proposed by: Jason Mao

Answer: $\boxed{8\sqrt{6}}$

Solution: Let A, B, and C be the centers of the three smaller circles, let the three chords be ℓ_1, ℓ_2, and ℓ_3, and let X, Y, and Z be the midpoints of sides $\overline{BC}$, $\overline{AC}$, and $\overline{AB}$, respectively, as shown below.



Since ℓ_1 is tangent to circles A and B of radius 2, it follows that $\ell_1 \parallel \overline{AB}$, and that the distance between ℓ_1 and $\overline{AB}$ is exactly 2. The same holds for ℓ_2 and $\overline{AC}$, as well as for ℓ_3 and $\overline{BC}$.

We may now compute $OY = OZ$ to be 2 more than the distance from O to ℓ_1, which is:

$$OY = OZ = 2 + \sqrt{11^2 - \left(\frac{1}{2} \cdot 4\sqrt{30}\right)^2} = 3.$$

Furthermore, since circle A of radius 2 is internally tangent to circle O of radius 11, it follows that $OA = 11 - 2 = 9$. Similarly, $OB = OC = 9$. Hence, O is the circumcenter of isosceles $\triangle ABC$.

Therefore, $\triangle AZO$ and $\triangle AYO$ are right, so the Pythagorean Theorem yields:

$$AZ = BZ = AY = CY = \sqrt{9^2 - 3^2} = 6\sqrt{2}.$$

We now compute OX by noting that $\triangle AZO \sim \triangle AXB$, so:

$$\frac{AZ}{AO} = \frac{AX}{AB} \implies \frac{6\sqrt{2}}{9} = \frac{OX + 9}{12\sqrt{2}} \implies OX = 7.$$

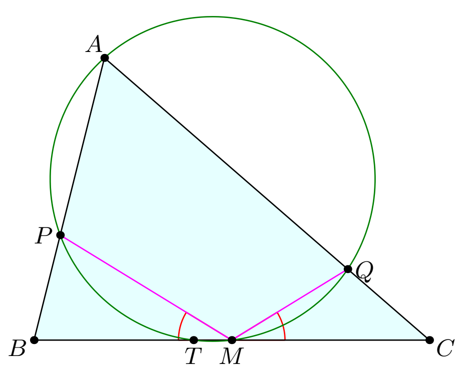
Thus, the distance from O to ℓ_3 is $OX - 2 = 5$, so the length of ℓ_3 is $2 \cdot \sqrt{11^2 - 5^2} = \boxed{8\sqrt{6}}$.

6. Let ABC be a triangle, and M be the midpoint of segment $\overline{BC}$. Points P and Q lie on segments $\overline{AB}$ and $\overline{AC}$, respectively, so that $\angle PMB = \angle QMC = \frac{1}{2}\angle BAC$. Given that $AP = 1$, $AQ = 3$, and $BC = 8$, compute BP.

   Proposed by: Jason Mao

   Answer: $\boxed{\sqrt{17} - 1}$

   Solution 1:



Claim 1. $BP = CQ$.

Proof. We have that

$$\begin{aligned}&\angle BPM + \angle MQC \\ &= 360^\circ - \angle MBP - \angle PMB - \angle CMQ - \angle QCM \\ &= 360^\circ - \angle CBA - \angle BAC - \angle ACB \\ &= 180^\circ.\end{aligned}$$

Thus by the Law of Sines,

$$BP = \frac{BM \cdot \sin \angle PMB}{\sin \angle BPM} = \frac{CM \cdot \sin \angle MQC}{\sin \angle CMQ} = CQ.$$

□

Let x be the length of BP, and let the circle $(APMQ)$ meets $\overline{BC}$ again at T. Then by Power of a Point,

$$x(x+1) + x(x+3) = BT \cdot 4 + CT \cdot 4 = 32.$$

Thus $x = \boxed{\sqrt{17} - 1}$.

Solution 2: Let T be the foot of the angle bisector of $\angle BAC$. Let a,b,c be the lengths BC, AC, AB respectively, we are given $a = 8$.

From the angle-bisector theorem, $\frac{BT}{CT} = \frac{c}{b}$. Since $BT + CT = a$, we get $BT = \frac{ac}{b+c}$. We also have that $\triangle BPM \sim \triangle BTA$ because $\angle MBP = \angle ABT$ and $\angle PMB = \angle TBA$. From these two, we calculate

$$AP = AB - BP = AB - \frac{BM \cdot BT}{BA} = c - \frac{a^2}{2(b+c)} = \frac{2bc + 2c^2 - a^2}{2(b+c)}.$$

Similarly, $AQ = \frac{2bc+2b^2-a^2}{2(b+c)}$. We can now solve for b and c. We have that

$$b - c = \frac{2b^2 - 2c^2}{2(b+c)} = AQ - AP = 2.$$

Thus, $b = c + 2$. Substituting $a = 8$ and $b = c + 2$ into the equation for AP, we get

$$\begin{aligned}\frac{2c(c+2) + 2c^2 - 64}{2(2c+2)} = 1 &\implies 4c^2 + 4c - 64 = 4c + 4 \\ &\implies c = \sqrt{17}.\end{aligned}$$

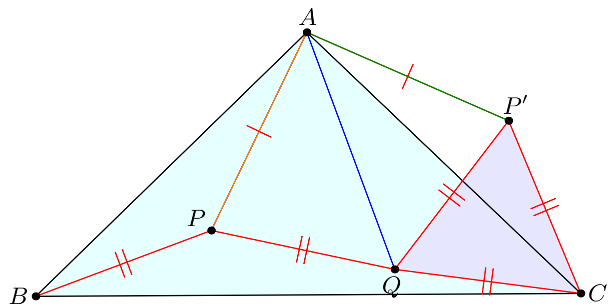
Hence $BP = AB - AP = c - 1 = \boxed{\sqrt{17} - 1}$.

7. Let ABC be an isosceles triangle with $AB = AC$. Points P and Q are located inside triangle ABC such that $BP = PQ = QC$. Suppose that $\angle BAP = 20^\circ$, $\angle PAQ = 46^\circ$, and $\angle QAC = 26^\circ$. Compute the measure of $\angle APC$.

   Proposed by: Andrew Brahms, Jackson Dryg, Jason Mao, Pitchayut Saengrungkongka

   Answer: $\boxed{74^\circ}$

   Solution 1:



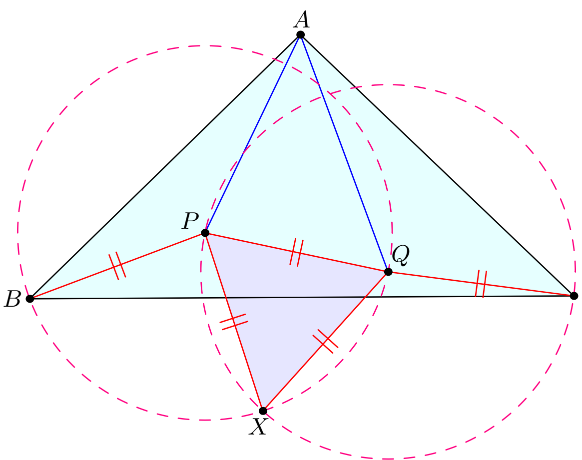
Let P' be the point outside $\triangle ABC$ such that $\triangle APB \cong \triangle AP'C$. Note that

- $\angle QAP' = \angle QAC + \angle CAP' = \angle QAC + \angle BAP = 20^\circ + 26^\circ = 46^\circ = \angle PAQ$.
- $AP = AP'$

Therefore, $\triangle APQ \cong \triangle AP'Q$, so $P'Q = PQ = QC$, which means that Q is the circumcenter of $\triangle PP'C$. Since $P'C = PB = QC$, we get that $\triangle QP'C$ is an equilateral triangle. By angle chasing,

$$\angle APC = \angle APP' + \angle P'PC = 90^\circ - \angle PAQ + \frac{1}{2} \cdot \angle P'QC = 90^\circ - 46^\circ + 30^\circ = \boxed{74^\circ}.$$

Solution 2:



Let X be the reflection of B across AP. Then, note that

- $AX = AB = AC$.
- $\angle XAC = \angle PAQ - \angle PAX = \angle PAQ - \angle PAB = 46^\circ - 20^\circ = 26^\circ = \angle QAC$.

Therefore, $\triangle AXQ \cong \triangle ACQ$, so X is the reflection of C across AQ too. Therefore, $PX = PB = PQ$ and $QX = QC = PQ$, so $\triangle PQX$ is an equilateral triangle.

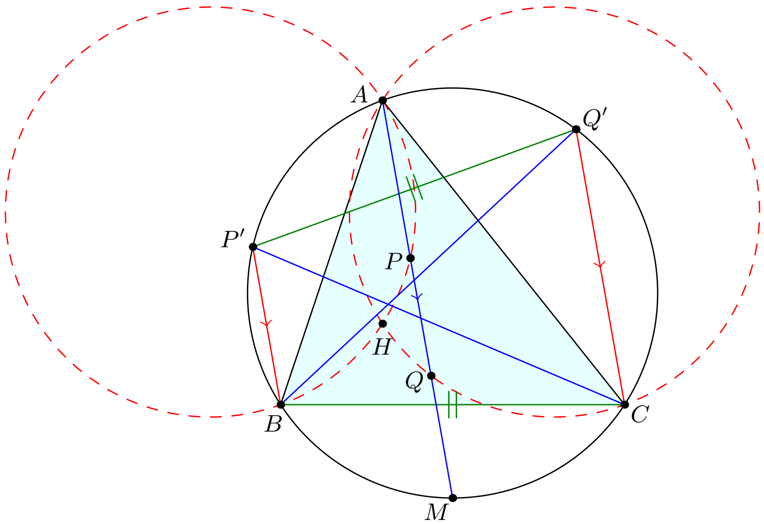
Note that Q is the circumcenter of $\triangle PXC$. Therefore, $\angle PCX = 30^\circ$. Because $XC \perp AQ$, we have $\angle ACX = 90^\circ - \angle QAC = 90^\circ - 26^\circ = 64^\circ$, so $\angle ACP = \angle ACX - \angle PCX = 34^\circ$. Finally, $\angle APC = 180^\circ - \angle ACP - \angle PAC = 180^\circ - 72^\circ - 34^\circ = \boxed{74^\circ}$.

8. Let ABC be a triangle with orthocenter H. The internal angle bisector of $\angle BAC$ meets the circumcircles of triangles ABH, ACH, and ABC again at points P, Q, and M, respectively. Suppose that points A, P, Q, and M are distinct and lie on the internal angle bisector of $\angle BAC$ in that order. Given that $AP = 4$, $AQ = 5$, and $BC = 7$, compute AM.

Proposed by: Pitchayut Saengrungkongka

Answer: $\boxed{\sqrt{69}}$

Solution 1:

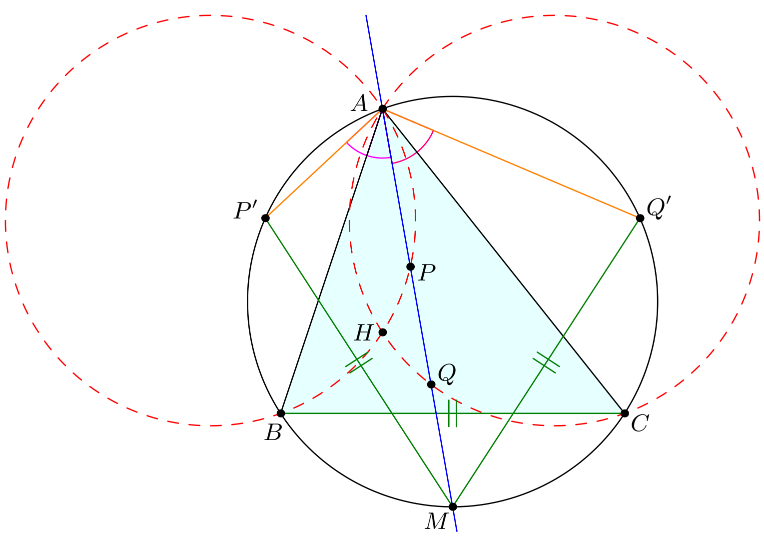


Because H is the orthocenter of $\triangle ABC$, the circumcircles of $\triangle ABH$ is the reflection of the circumcircle of $\triangle ABC$ across the midpoint of AB. Let P' be the reflection of P across the midpoint of AB, then P' must lie on the circumcircle of $\triangle ABC$. Similarly, let Q' be the reflection of Q across the midpoint of AC, then Q' must lie on the circumcircle of $\triangle ABC$ too.

Note that $BP' = 4$, $CQ' = 5$. Moreover, $P'B \parallel AM \parallel Q'C$, so $P'Q' = BC = 7$. Furthermore, since $\angle BCQ' = \angle ACM$, we have $AM = BQ'$. Similarly, we also have $AM = CP'$.

By Ptolemy theorem on the cyclic quadrilateral $BCQ'P'$, we have that $BP' \cdot CQ' + BC \cdot P'Q' = BQ'\dot{C}P'$. Substituting the side lengths back gives $4 \cdot 5 + 7 \cdot 7 = AM^2$, so $AM = \boxed{\sqrt{69}}$.

Solution 2:



Instead of reflecting across the midpoints of AB, AC, the reflection across the lines AB, AC also works too.

Because H is the orthocenter of $\triangle ABC$, the circumcircle of $\triangle ABH$ is the reflection of the circumcircle of $\triangle ABC$ across lines AB. Let P' be the reflection of P across line AB, then it must lie on the circumcircle of $\triangle ABC$. Similarly, let Q' be the reflection of Q across line AC, then it must lie on the circumcircle of $\triangle ABC$ too.

We note that $AP' = 4$ and $AQ' = 5$. Moreover, $\angle P'AB = \angle BAM = \angle MAC = \angle CAQ'$, so $\angle P'AM = \angle BAC = \angle MAQ'$. As a result, $P'M = BC = MQ' = 7$.

Now, we can only focus on quadrilateral $AP'Q'M$, which we know all side lengths. There are a couple ways to find the length AM.

- Let T be the intersection of lines AM and $P'Q'$. Note that
  (a) Since $\triangle MTP' \sim \triangle MP'A$, we have $MT \cdot MA = MP'^2 = 49$. (This is known as "shooting lemma".)
  (b) Since $\triangle AP'T \sim \triangle AMQ'$, we have $AT \cdot AM = AP' \cdot AQ' = 20$. (This is known as $\sqrt{AP' \cdot AQ'}$-inversion.)

  Adding these two equations together implies that $69 = MT \cdot MA + AT \cdot AM = AM(MT + AT) = AM^2$. Therefore, the length of AM is $\boxed{\sqrt{69}}$.
- Using the Law of Cosines on $\triangle AP'M$ and $\triangle AQ'M$,

$$AM^2 = AP'^2 + P'M^2 - 2AP' \cdot P'M \cos\angle AP'M = 65 - 56\cos\angle AP'M$$
$$AM^2 = AQ'^2 + Q'M^2 - 2AQ' \cdot Q'M \cos\angle AQ'M = 74 - 70\cos\angle AQ'M$$

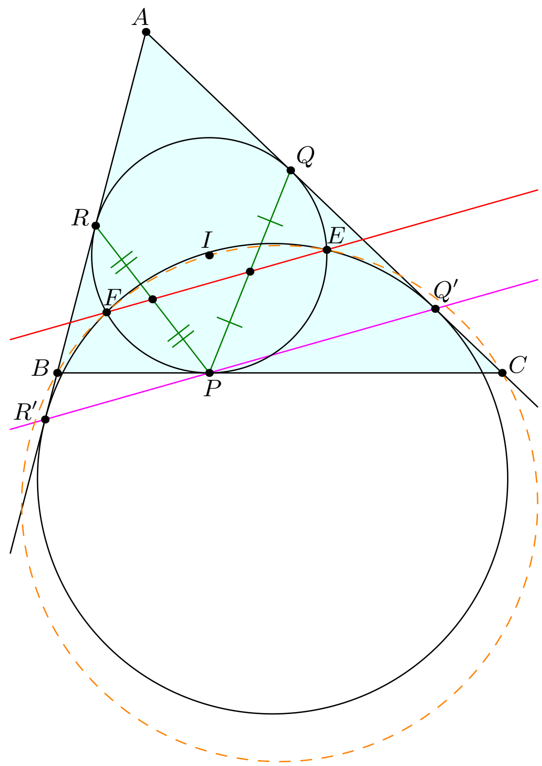
  Moreover, since $\angle AP'M + \angle AQ'M = 180^\circ$, we get that $\cos\angle AP'M + \cos\angle AQ'M = 0^\circ$. Solving the two equations above gives us that the length of AM is $\boxed{\sqrt{69}}$.

9. Let ABC be triangle with incenter I and incircle ω. The circumcircle of triangle BIC intersects ω at points E and F. Suppose that $\Gamma \neq \omega$ is a circle passing through E and F and tangent to lines AB and AC. If $AB = 5$, $AC = 7$, and Γ has twice the radius of ω, compute BC.

   Proposed by: Aprameya Tripathy

   Answer: $\boxed{3 + \sqrt{11}}$

   Solution:



Let P, Q, and R be the points the incircle touches BC, CA, and AB respectively. Note that by the radical axis theorem on $(CPIQ)$, (BIC), and ω, we have that CI, EF and PQ are concurrent. Hence, the midpoint of PQ lies on the EF. Similarly, by the radical axis theorem on $(BPIR)$, (BIC), and ω, the midpoint of PR lies on EF

Let Q' and R' be the points where Γ touches AB and AC respectively. Since Γ has twice the radius of ω, we have Q and R are midpoints of AQ' and AR' respectively. However, since midpoints of QQ' and RR' lie on the radical axes on ω and Γ, which is EF, we get that EF is equidistant to lines QR and $Q'R'$. Note that EF is also the P-midline of $\triangle PQR$, so P lies on $Q'R'$.

We finish with simple length chasing. Let $BC = x$. Then $BP = \frac{x}{2} - 1$, $CP = \frac{x}{2} + 1$, and

$$AQ' = AR' = 2 \cdot AQ = 2\left(6 - \frac{x}{2}\right) = 12 - x.$$

Thus, by Menelaus's Theorem,

$$1 = \frac{AR'}{R'B} \cdot \frac{BC}{PC} \cdot \frac{CQ'}{Q'A} = \frac{12 - x}{7 - x} \cdot \frac{x - 2}{x + 2} \cdot \frac{x - 5}{12 - x}.$$

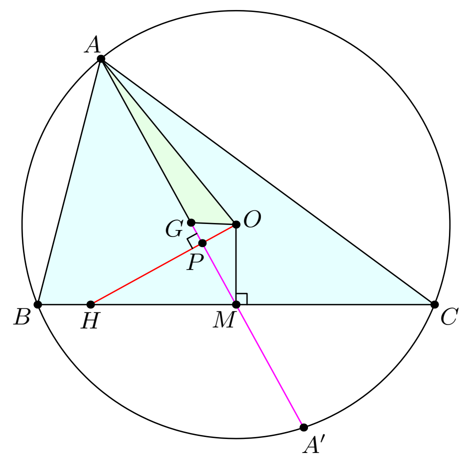
Simplifying, we $x^2 - 6x - 2 = 0$, which gives us $BC = \boxed{3 + \sqrt{11}}$

10. Let ABC be a triangle with centroid G and circumcenter O. Suppose that the orthocenter of triangle AGO lies on line BC. Given that $AB = 11$ and $AC = 13$, compute BC.

Proposed by: Aprameya Tripathy

Answer: $\boxed{\sqrt{\frac{580}{3}} = \frac{\sqrt{1740}}{3}}$

Solution 1:



Let H be the orthocenter of $\triangle AGO$ and P be the foot of altitude from O to AG. Let M be the midpoint of BC. Notice that

- Since $\triangle POM \sim \triangle PMH$, we get that $PM^2 = PO \cdot PH$.
- Since $\triangle POG \sim \triangle PAH$, we get that $PO \cdot PH = PG \cdot PA$.

Thus, we get that $PM^2 = PG \cdot PA$.

Note that $3MG = AM$ implies that $3(PM + PG) = PA + PM$ and $2PM = PA - 3PG$. Substituting back to get that $4PG \cdot PA = 4PM^2 = (PA - 3PG)^2$ and $0 = PA^2 - 10PG \cdot PA + 9PG^2$. Hence, $0 = \left(\frac{PA}{PG}\right)^2 - 10\left(\frac{PA}{PG}\right) + 9$. Solving this quadratic equation, we get that $PA = PG$ or $PA = 9PG$. Clearly, $PA \neq PG$, so $PA = 9PG$. From $PM^2 = PG \cdot PA$, we have $AP = 3PM$ too.

Let $AP = 3\ell$ and $PM = \ell$. Let AM intersect the circumcircle again at A'. Because OP is perpendicular to AA', P is the midpoint of AA', we have $AA' = 6\ell$, so $MA' = 6\ell - 4\ell = 2\ell$. By power of point, we have

$$MB^2 = MB \cdot MC = MA \cdot MA' = (4\ell)(2\ell) = 8\ell^2 \implies MB^2 = \sqrt{8}\ell.$$

Thus, the length of median formula gives

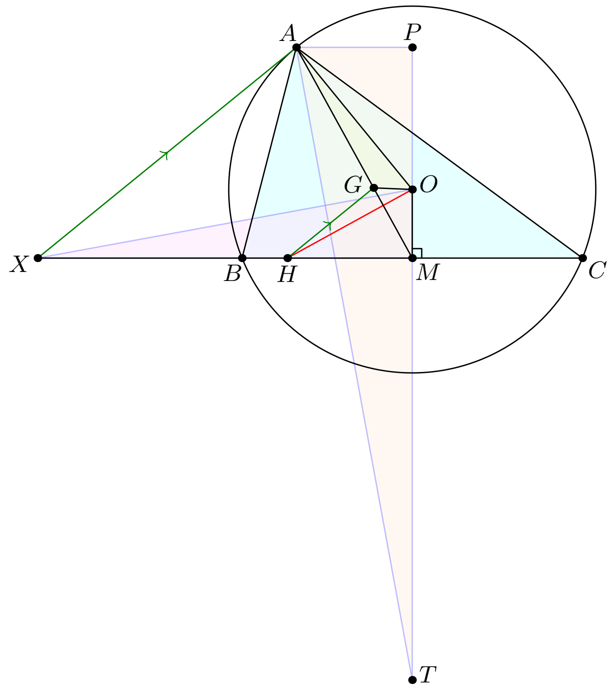
$$AB^2 + AC^2 = 2(AM^2 + MB^2) = 2(16\ell^2 + 8\ell^2) = 48\ell^2,$$

which implies that $\ell^2 = \frac{AB^2 + AC^2}{48}$. Hence,

$$BC^2 = 4BM^2 = 32\ell^2 = \tfrac{2}{3}(AB^2 + AC^2) = \tfrac{2}{3}(11^2 + 13^2) = \frac{580}{3},$$

which implies $BC = \boxed{\sqrt{\frac{580}{3}}}$.

Solution 2:



Let ω be the circumcircle of $\triangle ABC$. Let H be the orthocenter of $\triangle AGO$, which lies on BC. Let X be the intersection of line BC and the tangent line to ω at A. Let T be the intersection of tangent lines to ω at B and C. Let P be the foot from A to OM. Observe the following:

- Since both GH and XA are perpendicular to AO, we get that $AX \parallel GH$ and $XH : HM = AG : GM = 2 : 1$.
- By La Hire's theorem, X is the pole of AT with respect to $\odot(ABC)$, so $OX \perp AT$.
- We have $\triangle OMX \cup H \sim \triangle APT \cup M$ because $OM \perp AP$, $OX \perp AT$, and $OH \perp AM$.

Hence, the first and third bullet point gives $TM : MP = XH : HM = 2 : 1$. In particular, $[BTC] = 2[BAC]$.

We now show two different ways to finish from here.

- Note that T has barycentric coordinate $(-2, x, y)$ for some real numbers x and y such that $x+y = 3$. However, the barycentric coordinate of T is well-known to be $(-a^2 : b^2 : c^2)$ (where $a = BC$, $b = CA$, and $c = AB$). This follows from Corollary 15 of Evan Chen's barycentric coordinate handout. Therefore, $\frac{b^2+c^2}{3} = \frac{a^2}{2}$, which implies that $a^2 = \frac{2}{3}(11^2 + 13^2) = \frac{580}{3}$, so $a = \boxed{\sqrt{\frac{580}{3}}}$.
- Note that $[BAC] = \frac{1}{2}bc\sin A$ and $MT = \frac{a}{2}\tan A$, which implies

$$[BTC] = \frac{1}{2} \cdot a \cdot \frac{a}{2}\tan A = \frac{a^2}{4}\tan A.$$

  Therefore, the condition $[BTC] = 2[BAC]$ translates to

$$bc\sin A = \frac{a^2}{4}\tan A$$
$$bc\cos A = \frac{a^2}{4}$$
$$\frac{b^2+c^2-a^2}{2} = \frac{a^2}{4}, \qquad \text{(Law of Cosines)}$$

  which implies that $a^2 = \frac{2}{3}(b^2+c^2) = \frac{2}{3}(11^2+13^2) = \frac{580}{3}$, so $a = \boxed{\sqrt{\frac{580}{3}}}$.