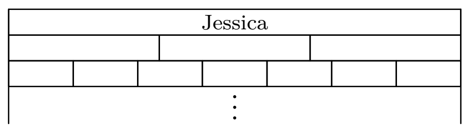
HMMT February 2026

February 14, 2026

Team Round

1. **[20]** Let $\mathcal{P}$ be a regular hexagon with center O. Distinct points A and B lie on the boundary of $\mathcal{P}$. The segments OA and OB divide the interior of $\mathcal{P}$ into two polygons $\mathcal{R}_1$ and $\mathcal{R}_2$. Let a_1 and a_2 be the areas of $\mathcal{R}_1$ and $\mathcal{R}_2$ respectively, and let p_1 and p_2 be the perimeters of $\mathcal{R}_1$ and $\mathcal{R}_2$ respectively. Given that $a_1/p_1 = a_2/p_2$, prove that $a_1 = a_2$.

2. **[25]** Jessica the jackrabbit wants to climb down a wall. The wall consists of 2026 horizontal layers stacked vertically. The nth layer from the top is partitioned into $2^n - 1$ identical rectangular bricks arranged side by side. Jessica begins in the topmost layer, which contains a single brick. A move consists of Jessica going down one layer to a brick that shares a side with the brick she is currently on.



Determine, with proof, the total number of distinct sequences of moves Jessica can take to reach the 2026th layer.

3. **[30]** Let α and β be complex numbers such that $\alpha\beta + \alpha + \beta + 100 = 0$. Suppose that $|\alpha| = |\beta| = M$ for some nonnegative real number M. Determine, with proof, all possible values of M.

4. **[35]** A set of rational numbers S is called *inclusive* if 0 is not an element of S, and for any (not necessarily distinct) elements x, y, and z of S, the number $xy + z$ is also an element of S. Determine, with proof, all rational numbers a for which there exists an inclusive set containing a.

5. **[40]** The numbers 1, 2, $\ldots$, 2026 are written on a blackboard. An operation consists of replacing any number on the blackboard with the positive difference between the largest and smallest numbers currently on the blackboard. Determine, with proof, the least number of operations required to make all the numbers on the blackboard equal.

6. **[40]** Let $ABCD$ be a cyclic quadrilateral with circumcenter O, and let M be the midpoint of $\overline{AB}$. Suppose that $\angle CMD = 90^\circ$ and $\overline{AD} \perp \overline{BC}$. Prove that the reflection of O over $\overline{AB}$ lies on line CD.

7. **[50]** An infinite sequence a_1, a_2, a_3, $\ldots$ of integers is called *r-close* if every integer appears in the sequence exactly once and $|a_{n+2} - a_{n+1}| \geq r|a_{n+1} - a_n|$ for all integers $n \geq 1$. Determine, with proof, all nonnegative real numbers r for which an r-close sequence exists.

8. **[50]** Let $n \geq 2$ be a positive integer and let $(a_1, \ldots, a_n)$ be a permutation of the numbers 1, 2, $\ldots$, n. Marin makes a single move on this permutation by performing the following steps:

   - First, he chooses an integer $1 \leq k \leq \lfloor n/2 \rfloor$. This integer can be different for different moves.
   - Then, he picks two nonintersecting sets $I = \{i_1, i_2, \ldots, i_k\}$ and $J = \{j_1, j_2, \ldots, j_k\}$ such that $1 \leq i_1 < i_2 < \cdots < i_k \leq n$ and $1 \leq j_1 < j_2 < \cdots < j_k \leq n$.
   - Finally, he swaps the numbers a_{i_s} and a_{j_s} in the current permutation for all integers $1 \leq s \leq k$.

   Let $f(n)$ denote the smallest positive integer such that it is possible for Marin to reach $(1, 2, \ldots, n)$ from any starting permutation in at most $f(n)$ moves. Prove that $\lceil \log_3 n \rceil \leq f(n) \leq \lceil \log_2 n \rceil$.

9. **[55]** Let ABC be an acute scalene triangle. Let D be the foot of the altitude from A to $\overline{BC}$, and let M be the midpoint of $\overline{BC}$. There exists a unique point P strictly inside triangle ABC such that $\angle DPM = 90^\circ$ and $PB/PC = AB/AC$. Prove that $\angle BPC = 180^\circ - |\angle ABC - \angle ACB|$.

10. **[55]** Prove that there exists a real constant M such that for every prime $p \geq M$ and any positive integer $2 \leq m \leq p - 1$, there exist positive integers a and b such that $m \leq a \leq 1.01m$, $p^{0.99} \leq b \leq p$, and p divides $ab - 1$.