

HMMT HMIC 2026

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1. [6] Let m and n be even positive integers. Suppose we have a tiling of an $m \times n$ rectangle with $mn/2$ non-overlapping dominoes (1×2 or 2×1). Prove there exists another tiling of the rectangle with dominoes that shares no dominoes with the original tiling.

(Two tilings are said to *share a domino* if there exists two cells of the rectangle which are covered by a single domino in both tilings.)

2. [7] Prove that there exist infinitely many positive integers n such that $n + 100$ divides $3^n - 2^n - 1$.
3. [8] Let ABC be a scalene triangle with circumcenter O and symmedian point K . Points X and Y lie on the interiors of sides AC and AB , respectively, such that $XY \parallel BC$. Suppose that the circumcircles of triangles AXB and AYC meet segment BC at points $D \neq B$ and $E \neq C$, respectively. Prove that D , E , X , and Y lie on a circle whose center lies on line OK .

(The symmedian point of triangle ABC is the intersection of the reflections of the B -median and C -median across the angle bisectors of $\angle ABC$ and $\angle ACB$, respectively.)

4. [10] Let $n \geq 4$ be an integer. Alice selects n pairwise distinct points A_1, A_2, \dots, A_n in the plane. In a single query, Bob selects four pairwise distinct integers i, j, k , and ℓ , each between 1 and n , inclusive, and sends them to Alice; Alice then responds “yes” if the lines A_iA_j and A_kA_ℓ are parallel or coincide, and “no” otherwise. In terms of n , determine the minimum number of queries Bob needs to determine whether all n points are collinear, regardless of how the points are positioned.
5. [11] Let n be a positive integer. Determine, in terms of n , the number of ordered n -tuples (a_1, a_2, \dots, a_n) of real numbers such that

- $0 \leq a_i < 1$ for all $1 \leq i \leq n$, and
- $a_1 \cdot 1^k + a_2 \cdot 2^k + \dots + a_n \cdot n^k$ is an integer for all nonnegative integers k .