HMMT November 2026

November 07, 2026

General Round

1. Each cell of the following 1×8 grid is labeled with a single-digit positive integer such that any two adjacent cells differ by either 3 or 5. Compute the sum of all eight numbers in the grid.

6	?	8	?	?	7	?	5

Proposed by: Jason Mao

Answer: $\boxed{38}$

Solution: Label the empty cells as follows:

6	a	8	b	c	7	d	5

Since a differs from 8 by 3 or 5, it must be equal to 3 or 5. However, since it also must differ from 6 by 3 or 5, we must have $a = 3$.

Similarly, since d differs from 7 by 3 or 5, it must be equal to 2 or 4, but since it also differs from 5 by 3 or 5, we must have $d = 2$.

Finally, note that by the same reasoning, b must be either 3 or 5, and c must be either 2 or 4. Since b and c must also differ by either 3 or 5, we must have $b = 5$ and $c = 2$.

Filling these empty cells in, we get the following grid:

6	3	8	5	2	7	2	5

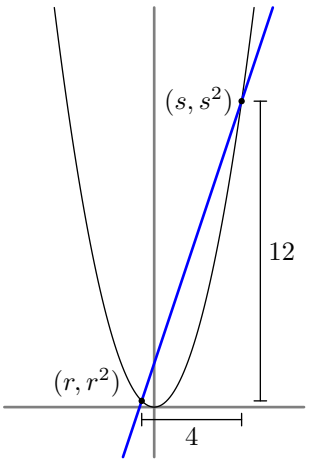
Adding the eight numbers together, we get an answer of $\boxed{38}$.

2. Suppose that m and b are positive real numbers such that the line $y = mx + b$ intersects the parabola $y = x^2$ at two points whose x-coordinates differ by 4 and whose y-coordinates differ by 12. Compute b.

Proposed by: Jackson Dryg

Answer: $\boxed{\frac{7}{4}}$

Solution 1: Let the intersection points be (r, r^2) and (s, s^2).



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By reflecting the parabola and line over the y-axis if necessary (which doesn't change b), we can assume without loss of generality that r is positive and has greater absolute value than s.

Then, the two conditions give us

$$r - s = 4,$$
$$r^2 - s^2 = 12.$$

Note that $r^2 - s^2 = (r - s)(r + s)$, so

$$r + s = \frac{r^2 - s^2}{r - s} = \frac{12}{4} = 3.$$

Adding this to $r - s = 4$ gives $2r = 7$, so $r = 7/2$. Substituting back in to $r - s = 4$ gives $s = -1/2$.

Additionally, because the line passes through the points (r, r^2) and (s, s^2), the slope is

$$m = \frac{r^2 - s^2}{r - s} = 3.$$

Hence the line is of the form $y = 3x + b$. Since $s = -1/2$, we have $(s, s^2) = (-1/2, 1/4)$. Substituting this point into the equation of the line, we obtain

$$\frac{1}{4} = 3\left(-\frac{1}{2}\right) + b \implies b = \frac{1}{4} - 3\left(-\frac{1}{2}\right) = \boxed{\frac{7}{4}}.$$

Solution 2: After solving for r and s, there is a quicker way to finish. Note that r and s are the roots of the quadratic $x^2 - mx - b$, so by Vieta,

$$b = -rs = -\left(\frac{-1}{2}\right)\left(\frac{7}{2}\right) = \boxed{\frac{7}{4}}.$$

3. Eric the monkey has no bananas. Every morning, Eric may optionally go to the supermarket to buy a single bunch of exactly 5 bananas. Eric may only go to the supermarket 3 times. Every night, Eric must eat 1 banana or else he dies. Compute the number of ways Eric may survive for 15 nights.

Proposed by: Jason Mao

Answer: $\boxed{35}$

Solution: The fact that Eric survives all 15 nights is equivalent to the following set of conditions:

- Eric goes to the supermarket on day 1, so that he has a banana to eat on the first night.
- Eric goes to the supermarket at least twice by night 6, since he can only get 5 bananas after one trip.
- Eric goes to the supermarket three times by night 11, since he can only get 10 bananas after two trips.

Notice that if these conditions are satisfied, Eric will have a banana to eat on each of the first 15 nights.

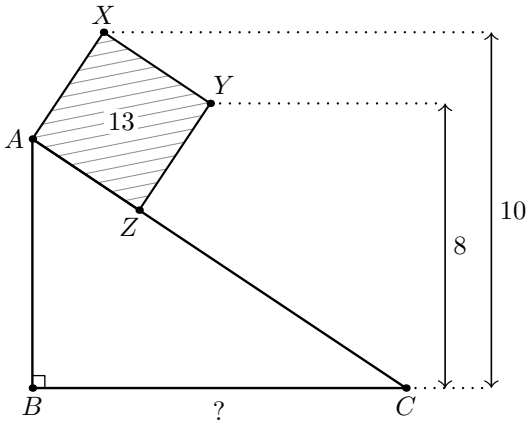
This means Eric completes 2 trips between days 2 and 11, inclusive. There are $\binom{10}{2}$ ways to pick two days within this range. However, Eric cannot make both of these trips between days 7 and 11, inclusive, since he needs to make his second trip by day 6. This eliminates $\binom{5}{2}$ choices.

Thus, the answer is

$$\binom{10}{2} - \binom{5}{2} = \boxed{35}.$$

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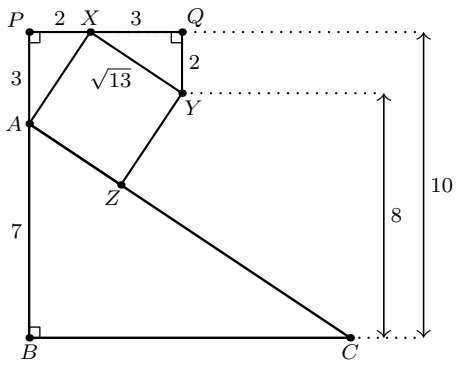
4. Square $AXYZ$ with area 13 is placed along hypotenuse $\overline{AC}$ of right triangle ABC. Given that the distances from X and Y to $\overline{BC}$ are 10 and 8 respectively, compute BC.



Proposed by: Jason Mao, Pitchayut Saengrungkongka

Answer: $\boxed{\frac{21}{2}}$

Solution: Construct points P and Q as shown below, so that $\angle XPA$ and $\angle YQX$ are right angles, and $\overline{PAB}$ and $\overline{PXQ}$ are collinear.



Note that the side length of the square is $\sqrt{13}$. Also, $QY = 10 - 8 = 2$. By the Pythagorean theorem,

$$QX = \sqrt{XY^2 - QY^2} = \sqrt{13 - 4} = 3.$$

Since $\triangle XPA$ and $\triangle YQX$ are congruent, we have $PX = 2$ and $PA = 3$, so

$$AB = 10 - 3 = 7.$$

Finally, observe that $\triangle XPA$ and $\triangle ABC$ are similar, so

$$BC = AP \cdot \frac{AB}{PX} = 3 \cdot \frac{7}{2} = \boxed{\frac{21}{2}}.$$

5. Let α be the unique real number such that both $\alpha + \sqrt{\frac{11}{12}}$ and $\alpha^3 + \sqrt{\frac{11}{12}}$ are positive and rational. Compute $\alpha^3 + \sqrt{\frac{11}{12}}$.

Proposed by: Jason Mao

Answer: $\boxed{\frac{25}{54}}$

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Solution: Let $q = \alpha + \sqrt{\frac{11}{12}}$, where q is a positive rational number. Notice that

$$\begin{aligned}\alpha^3 + \sqrt{\frac{11}{12}} &= \left(q - \sqrt{\frac{11}{12}}\right)^3 + \sqrt{\frac{11}{12}} \\ &= \left(q^3 - 3q^2 \cdot \sqrt{\frac{11}{12}} + 3q \cdot \frac{11}{12} - \frac{11}{12} \cdot \sqrt{\frac{11}{12}}\right) + \sqrt{\frac{11}{12}} \\ &= \left(q^3 + \frac{11}{4}q\right) + \left(-3q^2 + \frac{1}{12}\right) \cdot \sqrt{\frac{11}{12}}.\end{aligned}$$

Since the above is rational and $q^3 + \frac{11}{4}q$ is rational, $\left(-3q^2 + \frac{1}{12}\right) \cdot \sqrt{\frac{11}{12}}$ must be rational. Hence, $-3q^2 + \frac{1}{12}$ must be 0. Since q is positive, q must be $\frac{1}{6}$. Thus, the answer is

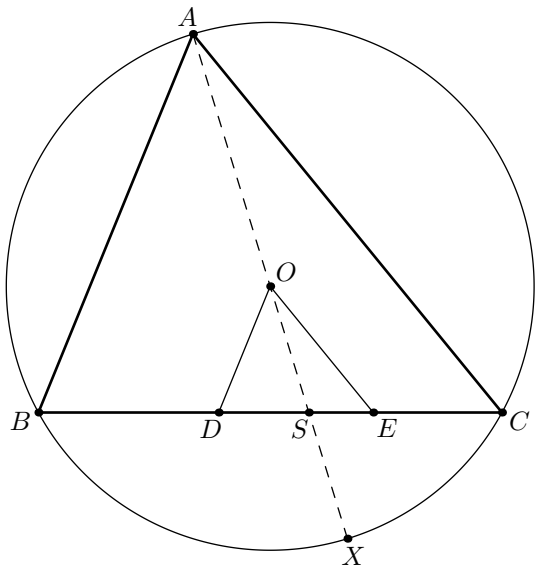
$$\alpha^3 + \sqrt{\frac{11}{12}} = q^3 + \frac{11}{4}q = \left(\frac{1}{6}\right)^3 + \frac{11}{4}\left(\frac{1}{6}\right) = \boxed{\frac{25}{54}}.$$

6. Let O be the circumcenter of an acute triangle ABC, and let D and E be points on side BC such that $\overline{OD}$ is parallel to $\overline{AB}$ and $\overline{OE}$ is parallel to $\overline{AC}$. Given that $BD = 7$, $DE = 6$, and $EC = 5$, compute AO.

Proposed by: Benny Wang

Answer: $\boxed{\sqrt{105}}$

Solution:



Let $R = AO$ be the circumradius of $\triangle ABC$. Let S and $X \neq A$ be the intersections of line AO with BC and the circumcircle of $\triangle ABC$ respectively.

Since $OD \parallel AB$ and $OE \parallel AC$,

$$\triangle SOD \sim \triangle SAB \text{ and } \triangle SOE \sim \triangle SAC.$$

Thus,

$$\frac{SD}{DB} = \frac{SO}{OA} = \frac{SE}{EC},$$

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which means

$$\frac{SD}{SE} = \frac{DB}{EC} = \frac{7}{5}.$$

Combining this with $SD + SE = 6$, we obtain $SD = \frac{7}{2}$ and $SE = \frac{5}{2}$. This implies

$$\frac{SO}{R} = \frac{SO}{OA} = \frac{SD}{DB} = \frac{7/2}{7} = \frac{1}{2} \implies SO = \frac{R}{2}.$$

Hence, $SX = OX - SX = R - \frac{R}{2} = \frac{R}{2}$. By power of a point at S,

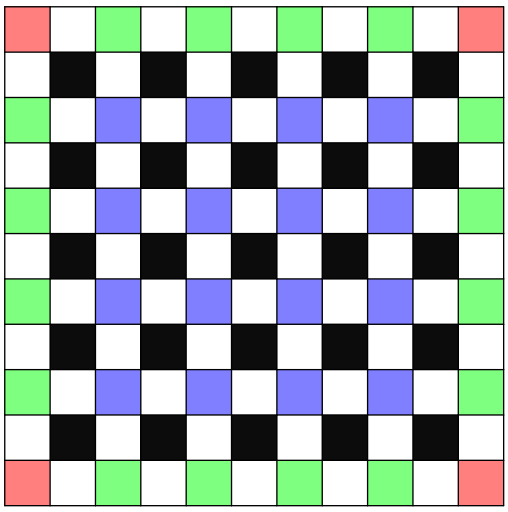
$$\begin{aligned} SA \cdot SX = SB \cdot SC &\implies (OA + OS) \cdot SX = \left(\frac{7}{2} + 7\right) \cdot \left(\frac{5}{2} + 5\right) \\ &\implies \left(\frac{3R}{2}\right) \cdot \frac{R}{2} = \frac{21}{2} \cdot \frac{15}{2} \\ &\implies R^2 = 105 \\ &\implies R = \boxed{\sqrt{105}}. \end{aligned}$$

7. In an 11×11 grid of unit squares, rows and columns are indexed consecutively with the integers from 1 to 11. Then, every cell with an even row and column index is deleted. Compute the number of ways to partition the resulting shape into rectangles with integer side lengths.

Proposed by: Benny Wang

Answer: $\boxed{3^4 \cdot 5^{16} \cdot 7^{16}}$

Solution 1: Call a grid cell *significant* if it has odd row and column index. Let's categorize the significant cells into the ones in the corners, edges, and middle.



A partition of the resulting shape into rectangles is equivalent to drawing some of the internal borders of the shape such that the shape is split into rectangles by these borders. For any partition, we call the resulting smaller shapes *regions*.

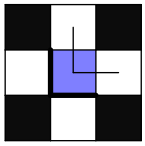
Call a significant cell *satisfied* at least one of the following is true:

- The left and right internal borders of the cell (of the ones that exist) are both drawn.
- The top and bottom internal borders of the cell (of the ones that exist) are both drawn.

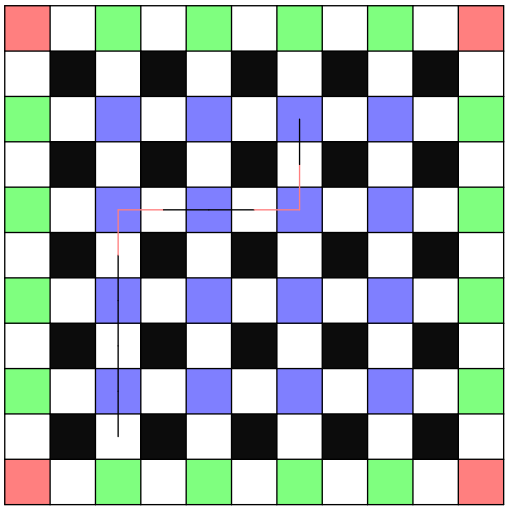
Claim 1. A choice of borders partitions the shape into rectangles if and only if every significant cell is satisfied.

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Proof. First, note that if a significant cell is not satisfied, then there is an L-shape of cells in the region including this significant cell. Since the last cell in the 2 by 2 rectangle containing this L-shape is not in the region, this region cannot be a rectangle.



Now suppose that every significant cell is satisfied. We claim that any two cells in the same region are in the same row or column. To do this, note that any two cells in the same region have a path connecting them through the cells of the shape that doesn't pass through any borders. This path must not turn, because it can only turn on a significant cell, but every significant cell is satisfied, making the path unable to turn.



Next, we show that all of the cells in the region are in the same row or column. Suppose that the region has at least 3 cells, since the region is trivially a rectangle if it has at most 2 cells. Take any two distinct cells A and B within the region, and assume without loss of generality that they are in the same row. Then any other cell C can't possibly be in the same column as both A and B, so it must be in the same row of at least one of A and B, meaning that it has to be in the same row as both of A and B. Therefore, all of the cells are in the same row, implying that the region is just a horizontal rectangle. (In the case that A and B are in the same column, we get a vertical rectangle instead.) □

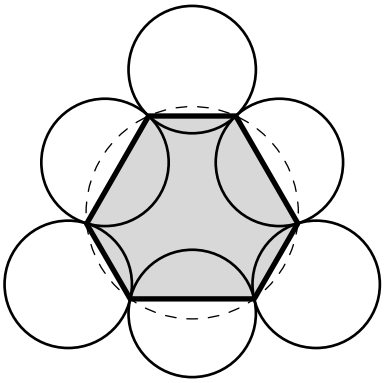
By the Principle of Inclusion-Exclusion, if a significant cell is next to a internal vertical borders and b internal horizontal borders, then the number of ways to satisfy the cell is exactly $2^a + 2^b - 1$. We also note that every internal border belongs to exactly one significant cell, so it suffices to multiply the number of ways to satisfy each significant cell.

- For each of the 4 corner significant cells, there are $2^1 + 2^1 - 1 = 3$ ways to make them satisfied.
- For each of the $4 \cdot 4 = 16$ edge significant cells, there are $2^2 + 2^1 - 1 = 5$ ways to make them satisfied.
- For each of the $4 \cdot 4 = 16$ middle significant cells, there are $2^2 + 2^2 - 1 = 7$ ways to make them satisfied.

Therefore, the answer is $\boxed{3^4 \cdot 5^{16} \cdot 7^{16}}$.

8. In the diagram below, six circles of radius 3 are arranged so that each is tangent to two others. Their tangency points form the vertices of a cyclic hexagon with circumradius 5. Compute the area of this hexagon.

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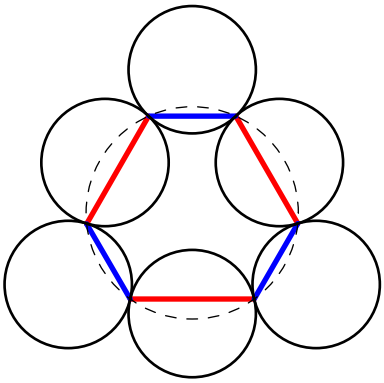
Proposed by: Jason Mao

Answer: $\boxed{\frac{147\sqrt{3}}{4}}$

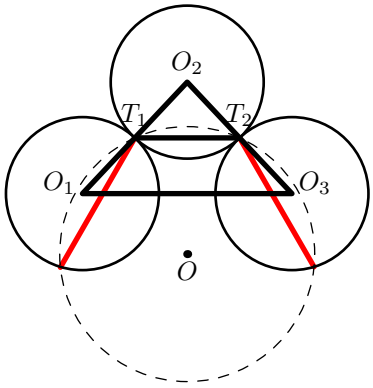
Solution: Label the centers of the six circles O_1, O_2, $\ldots$, O_6 in order, and the tangency points T_1, T_2, $\ldots$, T_6 in order, so that the circles with centers O_i and O_{i+1} are tangent at T_i (with $O_7 = O_1$). Let O denote the center of the circle through all six tangency points.

We first make some observations about the diagram.

Claim 1. All three red segments in the following diagram have equal length, as do all three blue segments.



Proof. Consider just the following following subset of the diagram:



Observe that $O_1O_2 = O_2O_3 = 2 \cdot 3 = 6$, so triangle $O_1O_2O_3$ is isosceles. As T_1 and T_2 are the midpoints of O_1O_2 and O_2O_3, respectively (since all circles have radius 3), $O_1T_1T_2O_3$ is an isosceles trapezoid. Thus, the perpendicular bisectors of T_1T_2 and O_1O_3 coincide. As $OT_1 = OT_2$, we conclude $OO_1 = OO_3$, so the red segments have equal length.

By similar reasoning, all three red segments have equal length and all three blue segments have equal length. □

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Claim 2. The hexagon $T_1T_2T_3T_4T_5T_6$ is equiangular, with every angle equal to $120°$.

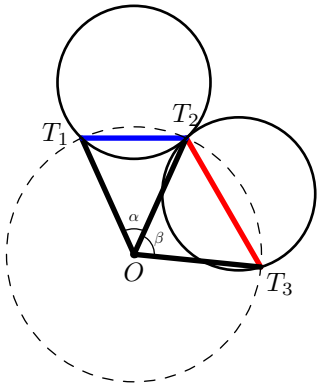
Proof. By the previous claim, let

$$\alpha = \angle T_1OT_2 = \angle T_3OT_4 = \angle T_5OT_6 \quad \text{and} \quad \beta = \angle T_2OT_3 = \angle T_4OT_5 = \angle T_6OT_1.$$

Then

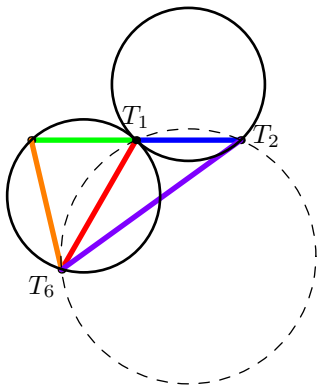
$$\angle T_1T_2T_3 = 180° - \frac{1}{2}\angle T_1OT_3 = 180° - \frac{1}{2}(\alpha+\beta),$$

as shown in the diagram below.



Similarly, every angle of the hexagon equals $180° - \frac{1}{2}(\alpha+\beta)$, so the hexagon is equiangular. As the six angles of a hexgaon sum to $720°$, each angle of this hexagon is $\frac{720°}{6} = 120°$. □

Say each red segment has length a, and each blue segment has length b. Extend one of the blue segments in the manner shown below.



Since the two solid circles are congruent, the blue and green segments have the same length (as they are reflections of each other across T_1).

By the previous claim, $\angle T_6T_1T_2 = 120°$. Thus, the orange segment subtends an arc of angle $120°$ in a circle of radius 3, so it has length $3\sqrt{3}$. Similarly, the purple segment subtends an arc of angle $120°$ in a circle of radius 5, so it has length $5\sqrt{3}$. Thus, by Law of Cosines on the red-green-orange and red-blue-purple triangles, we get that

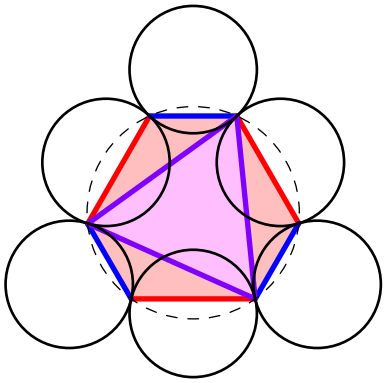
$$a^2 + b^2 + ab = (5\sqrt{3})^2 = 75 \quad \text{and} \quad a^2 + b^2 - ab = (3\sqrt{3})^2 = 27.$$

Subtracting the equations yields $2ab = 48$, so $ab = 24$. Then, the area of the red-blue-purple triangle $T_6T_1T_2$ is

$$\frac{1}{2} \cdot ab \sin(120°) = \frac{1}{2} \cdot 24 \cdot \frac{\sqrt{3}}{2} = 6\sqrt{3}.$$

Finally, note the hexagon can be split into an equilateral triangle of side length $5\sqrt{3}$ and the three congruent triangles $\triangle T_6T_1T_2$, $\triangle T_2T_3T_4$, and $\triangle T_4T_5T_6$, as shown below.

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Thus, the total area of the hexagon is

$$\frac{(5\sqrt{3})^2\sqrt{3}}{4} + 3 \cdot 6\sqrt{3} = \boxed{\frac{147\sqrt{3}}{4}}.$$

9. Let $a_1, a_2, \ldots$ be a sequence of positive integers such that $a_1 = a_2 = \cdots = a_7 = 1$ and

$$a_n = a_{n-1} + 10a_{n-7} \quad \text{for all } n \geq 8.$$

Compute the remainder when a_{2026} is divided by 10000.

Proposed by: Pitchayut Saengrungkongka

Answer: $\boxed{2991}$

Solution: We show by induction on n that

$$a_n = 1 + 10^1\binom{n-7}{1} + 10^2\binom{n-13}{2} + 10^3\binom{n-19}{3} + 10^4\binom{n-25}{4} + \cdots$$

where the top numbers in the binomial coefficients form an arithmetic progression with common difference -6. (By convention, $\binom{a}{b} = 0$ whenever $a < b$.)

Base cases: the cases $n \in \{1, 2, \ldots, 7\}$ are clear; since $n - 7 \leq 0$, only the first term in the sum is nonzero, yielding $a_n = 1$.

Induction step: Assume a_{n-1} and a_{n-7} both satisfy the formula above. Then,

$$\begin{aligned} a_{n-1} &= 1 + 10^1\binom{n-8}{1} + 10^2\binom{n-14}{2} + 10^3\binom{n-20}{3} + 10^4\binom{n-26}{4} + \cdots \\ 10a_{n-7} &= \quad 10^1\binom{n-8}{0} + 10^2\binom{n-14}{1} + 10^3\binom{n-20}{2} + 10^4\binom{n-26}{3} + \cdots \end{aligned}$$

Therefore,

$$\begin{aligned} a_n &= a_{n-1} + 10a_{n-7} \\ &= 1 + 10^1\binom{n-7}{1} + 10^2\binom{n-13}{2} + 10^3\binom{n-19}{3} + 10^4\binom{n-25}{4} + \cdots, \end{aligned}$$

where we add each term using Pascal's identity. This completes the induction.

Thus, for all positive integers n,

$$a_n = 1 + 10^1\binom{n-7}{1} + 10^2\binom{n-13}{2} + 10^3\binom{n-19}{3} + 10^4\binom{n-25}{4} + \cdots.$$

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Note that all but the first four terms of this sum are divisible by 10000, so to compute $a_{2026} \bmod 10000$, it suffices to compute the first four terms, i.e.,

$$1 + 10^1 \cdot (2026 - 7) + 10^2 \cdot \binom{2026 - 13}{2} + 10^3 \binom{2026 - 19}{3} \bmod 10^4.$$

Since $2026 - 13 = 2013 \equiv 13 \pmod{200}$, we have $2013 \cdot 2012 \equiv 13 \cdot 12 \pmod{200}$, so

$$\binom{2013}{2} \equiv \binom{13}{2} = 78 \pmod{100}.$$

Analogously, note $2026 - 19 = 2007 \equiv 7 \pmod{20}$, so $2007 \cdot 2006 \cdot 2005 \equiv 7 \cdot 6 \cdot 5 \pmod{20}$. As 3 is invertible modulo 20, we also have $\frac{1}{3}(2007 \cdot 2006 \cdot 2005) \equiv \frac{1}{3}(7 \cdot 6 \cdot 5) \pmod{20}$. Dividing both sides by 2,

$$\binom{2007}{3} \equiv \binom{7}{3} = 35 \equiv 5 \pmod{10}.$$

Thus, the answer is

$$1 + 190 + 7800 + 5000 \equiv \boxed{2991} \pmod{10000}.$$

10. The country of HMMTland has 100 cities. Engineers propose a tunnel between every pair of cities and randomly assign the distinct construction costs $1, 2, \ldots, \binom{100}{2}$ to the proposed tunnels, with every assignment equally likely. Each city funds its single cheapest incident tunnel. A tunnel is built if at least one city funds it. If more than one city funds it, it is still built only once. Compute the expected number of connected components in the resulting tunnel system.

(A *connected component* is a maximal nonempty set of cities such that every city in the set is reachable from every other city in the set by traveling through built tunnels.)

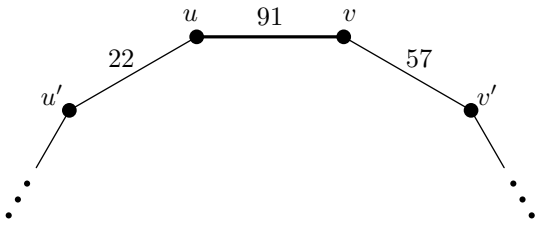
Proposed by: Bryan Guo

Answer: $\boxed{\frac{4950}{197}}$

Solution: We make the following observation about the tunnel system.

Claim 1. The tunnel system cannot contain a cycle; that is, it must be a forest of trees.

Proof. Assume for the sake of contradiction that the tunnel system does contain a cycle. Consider the most expensive tunnel this cycle, and say it connects cities u and v. Furthermore, say $u' \neq v$ is the other neighbor of u in this cycle, and $v' \neq u$ is the other neighbor of v in this cycle.



Then city u cannot fund the tunnel connecting u and v, because the tunnel connecting u and u' has lower cost than the tunnel connecting u and v. By similar reasoning, city v cannot fund the tunnel connecting u and v. So the tunnel is funded by neither u nor v, implying that it may not be built, contradiction. □

Call a tunnel *mutual* if both of its endpoint cities fund it, and say the resulting tunnel system has M built mutual tunnels. Then the resulting tunnel system has $100 - M$ tunnels. Since the tunnel system is a forest of trees, it follows that the tunnel system has exactly $100 - (100 - M) = M$ connected components. Therefore, it suffices to compute the expected value of M.

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Claim 2. For any two cities u and v, the probability that the proposed tunnel between u and v is both built and mutual in the final tunnel system is $\frac{1}{197}$.

Proof. Consider the set of all tunnels that are incident to at least one of u or v. There are 98 such tunnels incident only to u, there are 98 such tunnels incident only to v, and there is 1 such tunnel incident to both u and v, so there are $98 + 98 + 1 = 197$ such tunnels in total.

The proposed tunnel between u and v is both built and mutual if and only if its construction cost is the least among the construction costs of all 197 such tunnels. Each of the 197 tunnels is equally likely to have the least construction cost, so the probability that the tunnel between u and v is built and mutual is $\frac{1}{197}$. □

There are $\binom{100}{2}$ proposed tunnels, and each such proposed tunnel has a $\frac{1}{197}$ probability of being built and mutual. Thus, by linearity of expectation, the expected number M of built mutual tunnels is

$$\binom{100}{2} \cdot \frac{1}{197} = \boxed{\frac{4950}{197}}.$$

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