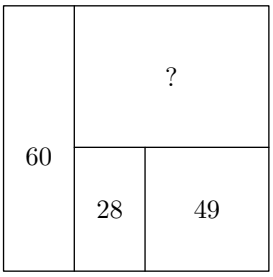
HMMT November 2026

November 07, 2026

Theme Round

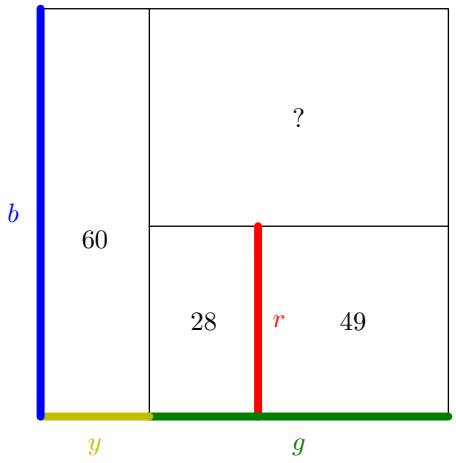
1. Moana is building a square raft by tying four rectangular planks together. All of the planks have integer side lengths, and three of the planks have areas 60, 28, and 49, as shown. Compute the area of the fourth plank.



Proposed by: Jackson Dryg, Jason Mao

Answer: $\boxed{88}$

Solution:



Let the lengths of the red, yellow, green, and blue segments shown in the diagram be r, y, g, and b, respectively.

The area of the rectangle with base g and r is $28 + 49 = 77$, so $r \cdot g = 77$. Furthermore, r must be an integer divisor of both 28 and 49, so it follows that r is either 1 or 7. If $r = 1$, then the length of the green segment is 77, and hence the side length of the square would be greater than 77. This is impossible because the side length of the square is equal to the blue segment, which must be an integer divisor of 60 and so must be less than 60. Hence $r = 7$ and $g = 11$.

From the left rectangle, we have $by = 60$. Because the entire raft is a square, $b = y + g = y + 11$. This implies that $y = b - 11$ and hence $b(b - 11) = 60$. We solve this quadratic:

$$b(b-11) = 60 \implies b^2 - 11b - 60 = 0 \implies (b-15)(b+4) = 0 \implies b = 15 \text{ or } b = -4.$$

Because b is positive, $b = 15$.

The dimensions of the fourth rectangle are $g = 11$ and $b - r = 15 - 7 = 8$. Hence, its area is $11 \cdot 8 = \boxed{88}$.

©2026 HMMT

2. Ariel is organizing her collection of human treasures. She can divide her dinglehoppers evenly into groups of 102, and her snarfblatts evenly into groups of 103. When she combines all of her dinglehoppers and snarfblatts, she can divide them evenly into groups of 100. Compute the minimum possible (nonzero) total number of dinglehoppers and snarfblatts in Ariel's collection.

Proposed by: Jackson Dryg

Answer: $\boxed{3500}$

Solution: Suppose that Ariel has $102x$ dinglehoppers and $103y$ snarfblatts for nonnegative integers x and y. Since she can combine all of her dinglehoppers and snarfblatts and divide them into groups of 100, we know that

$$102x + 103y \equiv 0 \pmod{100} \quad \Longrightarrow \quad 2x + 3y \equiv 0 \pmod{100}.$$

We want to minimize

$$102x + 103y = 100(x + y) + (2x + 3y).$$

As $2x + 3y$ is divisible by 100, it must be at least 100. Now observe that

$$x + y = \frac{1}{2}((2x + 3y) - y),$$

so if $2x + 3y$ is fixed, then $x + y$ is minimzed by making y as large as possible. If $2x + 3y = 100$, then y can be at most 32, yielding the solution

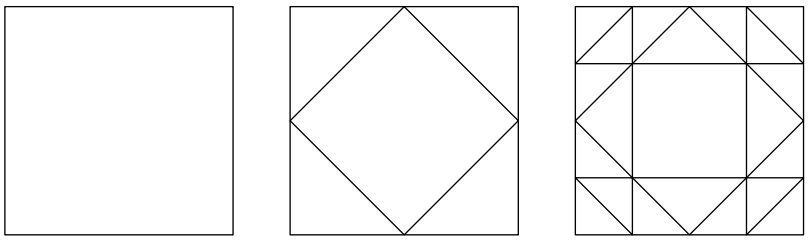
$$(x, y) = (2, 32).$$

Plugging this in yields that Ariel has at least

$$100(x + y) + (2x + 3y) = 100(2 + 32) + 100 = 35 \cdot 100 = \boxed{3500}$$

total dinglehoppers and snarfblatts.

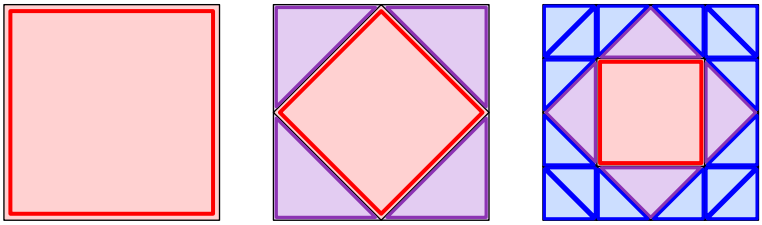
3. Jasmine is making a new design for a flying carpet. Her first draft is a square, and to construct her $(n + 1)$-th draft she takes her n-th draft and, for each polygonal region bounded by segments in her diagram, she connects the midpoints of adjacent sides via line segments. The first, second, and third drafts are shown below. Compute the number of polygonal regions in her 6th draft.



Proposed by: Kyle Wu

Answer: $\boxed{1365}$

Solution: Note that the second draft can be considered as 4^1 outer triangles and a rotated copy of the first draft. Subsequently, for every positive integer n, the $(n + 1)^{\text{th}}$ draft splits each of the outer triangles into 4 and makes the copy in the center into a copy of the n^{th} draft.



Draft 1 Draft 2 Draft 3

©2026 HMMT

In particular, the number of outer triangles multiplies by 4 with each draft, with there being $4 = 4^1$ of them on the second draft. Thus, if a_n denotes the number of polygonal regions in the n^{th} draft, we find that $a_{n+1} = a_n + 4^n$. As $a_1 = 1$, we compute

$$\begin{aligned} a_6 &= a_5 + 4^5 \\ &= a_4 + 4^4 + 4^5 \\ &= \cdots \\ &= a_1 + 4^1 + 4^2 + 4^3 + 4^4 + 4^5 \\ &= 4^0 + 4^1 + 4^2 + 4^3 + 4^4 + 4^5 \\ &= \frac{4^6 - 1}{3} = \frac{4095}{3} = \boxed{1365}. \end{aligned}$$

4. After some recent incidents, Snow White has decided to start eating apples again. She eats 1 apple on day 1. For each positive integer $n \geq 2$, she eats n times as many apples on day n as she did on all previous days combined. Compute the number of apples Snow White eats on day 2026.

   Proposed by: Derek Liu

   Answer: $\boxed{1013 \cdot 2026!}$

   Solution: Let a_n be the total number of apples Snow White eats in the first n days. Then,

   $$a_n = a_{n-1} + na_{n-1} = (n+1)a_{n-1},$$

   so

   $$a_n = (n+1)(n)(n-1)\cdots(3)a_1 = \frac{(n+1)!}{2}.$$

   Snow White eats $2026a_{2025}$ apples on day 2026, so the answer is

   $$2026 \cdot \frac{2027!}{2} = \boxed{1013 \cdot 2026!}.$$

5. Rapunzel's hair is getting too long again, so she decides to cut it every day for the next 6 days. At the start of the first day, her hair is 2026 feet long. First, she chooses a positive integer m. Then, on the n^{th} day, she cuts $\frac{m}{n}$ feet off her hair, then cuts $\frac{1}{n+1}$ of her remaining hair length off. Suppose that Rapunzel's hair does not grow back, and that at the end of each of the 6 days, her hair length is an integer number of feet. Compute the smallest possible value of m.

   Proposed by: Derek Liu

   Answer: $\boxed{74}$

   Solution: We claim that after day n, Rapunzel's hair length is

   $$\frac{2026 + m}{n+1} - m.$$

   We proceed via induction on n.

   Base case: $n = 0$. Her hair length starts at

   $$2026 = \frac{2026 + m}{0+1} - m$$

©2026 HMMT

regardless of m.

Inductive step: suppose that after day $n-1$, her hair length is

$$\frac{2026+m}{n}-m.$$

Then, after day n, her hair length is

$$\left(\frac{n}{n+1}\right)\left(\frac{2026+m}{n}-m-\frac{m}{n}\right)=\left(\frac{n}{n+1}\right)\left(\frac{2026-mn}{n}\right)=\frac{2026+m}{n+1}-m.$$

Thus, the claim is proved.

A value of m works if and only if this quantity is an integer for all n from 1 through 6, inclusive. As m is an integer, this is equivalent to $2026+m$ being divisible by every integer from 2 through 7. The least common multiple of these integers is $2^2\cdot 3\cdot 5\cdot 7=420$, so $420 \mid 2026+m$. The smallest multiple of 420 more than 2026 is $2100=2026+74$, so $m\geq \boxed{74}$.

6. Raya is flying on the dragon Sisu over Kumandra when they spot a small circular island they want to reach. Initially, Raya and Sisu are directly above the point $(0,0)$. The island consists of all points (x,y) satisfying

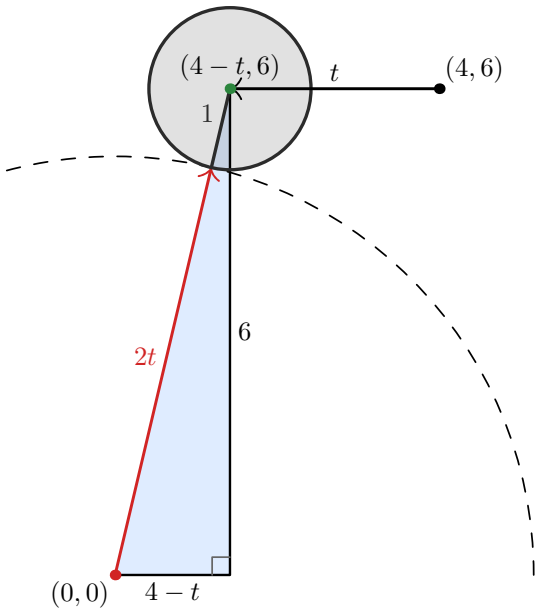
$$(x-4)^2+(y-6)^2\leq 1.$$

Without wind, Sisu can fly at 2 units per hour in any direction. Additionally, the wind blows Sisu in the positive x-direction at a constant rate of 1 unit per hour. Compute the minimum amount of time, in hours, before Raya and Sisu can be directly above any point on the island. (Assume that they remain at a constant altitude for the entirety of their flight.)

Proposed by: Darren Han, Liam Reddy

Answer: $\boxed{\sqrt{21}-2}$

Solution:



Observe that we can equivalently formulate the problem by removing the wind and instead having the island move in the negative x-direction at a constant rate of 1 unit per hour. In this formulation, after t hours, the island is centered at the point $(4-t,6)$. Also, Raya can travel a distance of at most $2t$

©2026 HMMT

and has reached the island if and only if she is within 1 unit of the island's center. Thus, Raya can reach the island if and only if $(4-t, 6)$ is within $2t+1$ units of the origin, i.e.,

$$\sqrt{(4-t)^2+6^2} \leq 2t+1.$$

Squaring and simplifying gives

$$3t^2+12t-51 \geq 0 \quad \Longrightarrow \quad t^2+4t-17 \geq 0.$$

Now, in order for a quadratic polynomial with positive leading coefficient to evaluate to a nonnegative number, the input must be either less than the smallest root or larger than the larger root. The two roots are

$$\frac{-4 \pm \sqrt{4^2-4(1)(-17)}}{2} = -2 \pm \sqrt{21}.$$

Since the smallest root $-2-\sqrt{21}$ is negative, we need

$$t \geq \boxed{\sqrt{21}-2}.$$

7. Mulan orders 10 soldiers to stand in a line. Each soldier points to one of their neighboring soldiers independently and uniformly at random. Compute the probability that exactly 3 pairs of neighboring soldiers point at each other.

Proposed by: Sebastian Attlan

Answer: $\boxed{\frac{63}{128}}$

Solution: Label a soldier pointing to the right with R, and a soldier pointing to the left with L. Note that the leftmost soldier is labeled R, while the rightmost soldier is labeled L.

A pair of soldiers pointing at each other corresponds to the substring 'RL'. If there are exactly 3 such pairs of soldiers, the labels can be grouped into 6 blocks:

$$\boxed{R\ldots R}\boxed{L\ldots L}\boxed{R\ldots R}\boxed{L\ldots L}\boxed{R\ldots R}\boxed{L\ldots L}$$

where each block has ≥ 1 soldier.

We count the number of such arrangements. This is equivalent to counting the number of ways to place 5 partitions into the 9 gaps between soldiers, where each partition represents a change of direction. For example,

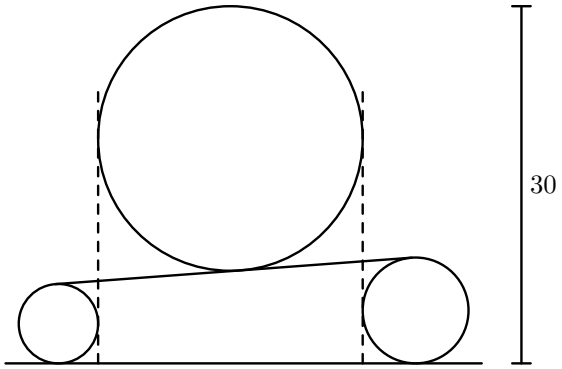
$$R\,|\,__\,|\,_\,|\,_\,|\,_\,|\,___\,L \quad \Longleftrightarrow \quad RLLRLRLLLL$$

Thus, there are $\binom{9}{5}$ such arrangements, out of 2^8 in total, giving an answer of

$$\frac{\binom{9}{5}}{2^8} = \frac{126}{256} = \boxed{\frac{63}{128}}.$$

8. Cinderella's pumpkin carriage consists of two circular wheels and a circular pumpkin, all tangent to a line segment. When on flat ground, the leftmost point of the pumpkin is directly above the rightmost point of the left wheel, and the rightmost point of the pumpkin is directly above the leftmost point of the right wheel. The wheels have radii 3 and 4. Moreover, the height of the carriage is 30. Compute the radius of the pumpkin.

©2026 HMMT



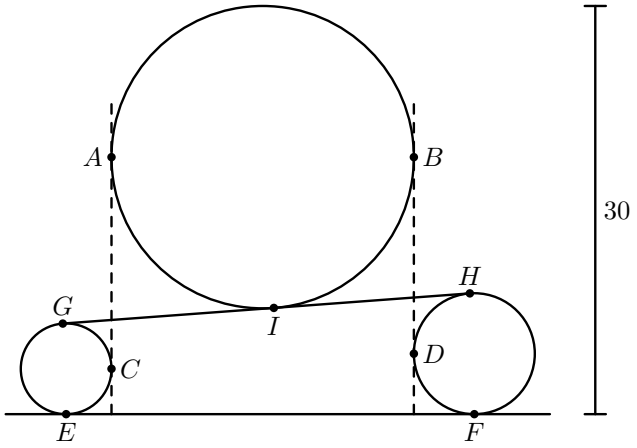
Proposed by: Benny Wang

Answer: $\boxed{\frac{23}{2}}$

Solution: Let circles ω_1, ω_2, and ω_3 denote the left wheel, pumpkin, and right wheel, respectively. Let their radii be $r_1 = 3$, r_2, and $r_3 = 4$ respectively. We also define the following points:

- Let A on ω_2 and C on ω_1 be on the vertical common tangent of ω_1 and ω_2.
- Let B on ω_2 and D on ω_3 be on the vertical common tangent of ω_3 and ω_2.
- Let E on ω_1 and F on ω_3 be on the horizontal common tangent of ω_1 and ω_3.
- Let G on ω_1 and H on ω_3 be on the other common tangent of ω_1 and ω_3.
- Let I on ω_2 lie on line GH.

See the diagram below.



The key observation is that, by repeatedly applying equal tangents,

$$AC + BD = GI + HI = GH = EF.$$

Because EF is horizontal and the circles are vertically aligned, $EF = r_1 + 2r_2 + r_3 = 2r_2 + 7$. Moreover, since AC is vertical and the total height of the carriage is 30, $r_1 + AC + r_2 = 30$, so $AC = 30 - r_1 - r_2 = 27 - r_2$. Similarly, $BD = 30 - r_2 - r_3 = 26 - r_2$. This implies that

$$2r_2 + 7 = (27 - r_2) + (26 - r_2) \implies r_2 = \boxed{\frac{23}{2}}.$$

©2026 HMMT

9. Compute the unique triple of positive integers (b, e, ℓ) satisfying both of the following equations:

$$be\ell + \ell + e = 6886,$$
$$be + \ell + \ell e = 586.$$

Proposed by: Derek Liu, Eric Wang

Answer: $\boxed{(19, 10, 36)}$

Solution: Subtracting the second equation from the first gives us

$$\begin{aligned} be\ell - be + e - \ell e &= e(b-1)(\ell-1) \\ &= 6300. \end{aligned}$$

Adding the two equations and adding 2 gives us

$$\begin{aligned} be\ell + be + e + \ell e + 2\ell + 2 &= ((b+1)e + 2)(\ell+1) \\ &= 7474 = 2 \cdot 37 \cdot 101. \end{aligned}$$

Thus, $\ell - 1 \mid 6300$ and $\ell + 1 \mid 7474$. Note that we need $\ell + 1 \geq 3$ so that $\ell - 1 > 0$. There are only six possible values for $\ell + 1 \geq 3$: they are 37, 74, 101, 202, 3737, 7474. Of these, only $\ell + 1 = 37$ makes $\ell - 1 = 35$ a divisor of 6300, so $\ell = 36$.

Then

$$e(b-1) = \frac{6300}{35} = 180$$

and

$$(b+1)e = \frac{7474}{37} - 2 = 200.$$

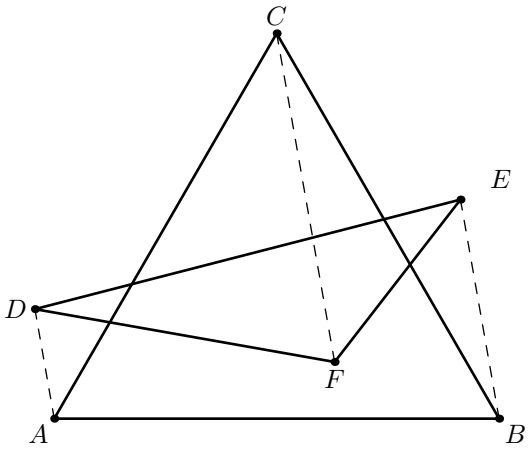
Subtracting these equations gives $2e = 20$, so $e = 10$ and $b = 19$. We conclude the unique solution is $(b, e, \ell) = \boxed{19, 10, 36}$.

10. Merida paints an equilateral triangle ABC with side length 4 units on her castle wall. She attempts to shoot an arrow at each vertex of this triangle. However, her arrows strike the wall at points D, E, and F, which are 1, 2, and 3 units away from A, B, and C, respectively. Furthermore, the centroid of triangle DEF coincides with the centroid of triangle ABC. Compute the maximum possible area of triangle DEF.

Proposed by: Aprameya Tripathy

Answer: $\boxed{4\sqrt{3} + 2\sqrt{21}}$

Solution 1:



©2026 HMMT

Let $\overrightarrow{X}$ denote the vector X, and $\overrightarrow{XY}$ denote the vector from X to Y. The centroid condition implies

$$\overrightarrow{AD} + \overrightarrow{BE} + \overrightarrow{CF} = (D - A) + (E - B) + (F - C) = 0.$$

Furthermore, $|\overrightarrow{AD}| = 1$, $|\overrightarrow{BE}| = 2$, and $|\overrightarrow{CF}| = 3$, so by the triangle inequality,

$$|\overrightarrow{AD} + \overrightarrow{BE} + \overrightarrow{CF}| \geq |\overrightarrow{CF}| - |\overrightarrow{BE}| - |\overrightarrow{AD}| = 3 - 2 - 1 = 0,$$

with equality when $\overrightarrow{AD}$, $\overrightarrow{BE}$, and $\overrightarrow{CF}$ are parallel with $\overrightarrow{CF}$ pointing in the opposite direction as $\overrightarrow{AD}$ and $\overrightarrow{BE}$. Thus, $\overrightarrow{BE} = 2\overrightarrow{AD}$ and $\overrightarrow{CF} = -3\overrightarrow{AD}$.

Now, the area of triangle DEF is just half the magnitude of the cross product

$$\begin{aligned}
\overrightarrow{DE} \times \overrightarrow{DF} &= (\overrightarrow{BE} - \overrightarrow{AD} + \overrightarrow{AB}) \times (\overrightarrow{CF} - \overrightarrow{AD} + \overrightarrow{AC}) \\
&= (\overrightarrow{AD} + \overrightarrow{AB}) \times (-4\overrightarrow{AD} + \overrightarrow{AC}) \\
&= \overrightarrow{AB} \times \overrightarrow{AC} + \overrightarrow{AD} \times (\overrightarrow{AC} - 4\overrightarrow{AB}) - 4\overrightarrow{AD} \times \overrightarrow{AD} \\
&= \overrightarrow{AB} \times \overrightarrow{AC} + \overrightarrow{AD} \times (\overrightarrow{AC} - 4\overrightarrow{AB}).
\end{aligned}$$

Note that $\overrightarrow{AD} \times (\overrightarrow{AC} - 4\overrightarrow{AB})$ has magnitude at most

$$|\overrightarrow{AD}| \cdot |\overrightarrow{AC} - 4\overrightarrow{AB}| = 1 \cdot |\overrightarrow{AC} - 4\overrightarrow{AB}| = \sqrt{4^2 + 16^2 + 2 \cdot \frac{1}{2} \cdot 4 \cdot 16} = 4\sqrt{21},$$

with equality achieved by making $\overrightarrow{AD}$ perpendicular to $\overrightarrow{AC} - 4\overrightarrow{AB}$. This is achievable because the direction of $\overrightarrow{AD}$ can be chosen arbitrarily. (By negating $\overrightarrow{AD}$, we can make the cross product have same direction as $\overrightarrow{AB} \times \overrightarrow{AC}$.)

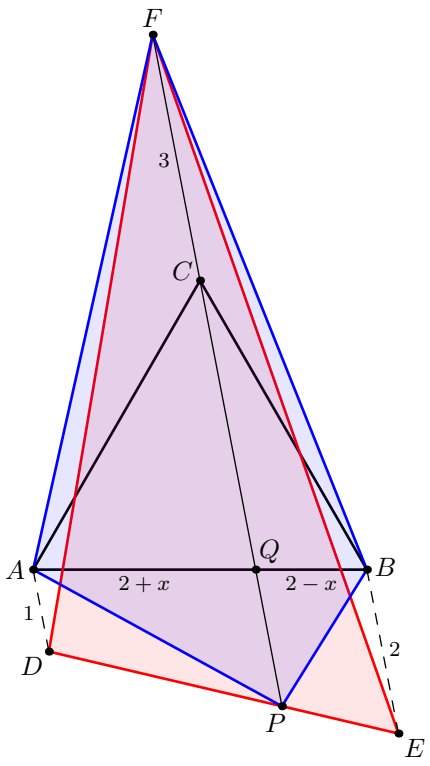
Also, $\overrightarrow{AB} \times \overrightarrow{AC}$ has magnitude equal to twice the area of triangle ABC, so it has magnitude $8\sqrt{3}$. Thus, noting that $\overrightarrow{AD} \times (\overrightarrow{AC} - 4\overrightarrow{AB})$ and $\overrightarrow{AB} \times \overrightarrow{AC}$ are parallel, we see that the area of DEF is

$$\left|\frac{1}{2}\overrightarrow{DE} \times \overrightarrow{DF}\right| \leq \frac{1}{2}(8\sqrt{3} + 4\sqrt{21}) = \boxed{4\sqrt{3} + 2\sqrt{21}},$$

which is achievable as noted above.

Solution 2: Similar to the first solution, we have that vectors $\overrightarrow{AD}$, $\overrightarrow{BE}$, and $\overrightarrow{CF}$ sum to 0, so they must form a triangle. Since these three vectors have lengths 1, 2, and 3, that triangle must be degenerate, implying that $AD \parallel BE \parallel CF$. We now present a different finish from here using only classical geometry.

©2026 HMMT



Let the line CF meet lines DE and AB at points P and Q. The key idea is the following area manipulation:

$$\begin{aligned}[DEF] &= [FDP] + [FEP] \\ &= [FBP] + [FAP] = [FABP] = [ABC] \cdot \frac{FP}{CQ},\end{aligned}$$

so it suffices to maximizes $\frac{FP}{CQ}$. To that end, let M be the midpoint of AB, and let $MQ = |x|$, where we assign a sign to x so that $x > 0$ if and only if A, M, Q lies in this order. In particular, $AQ = 2 + x$ and $BQ = 2 - x$. Then by Pythagorean's theorem, we have $CQ = \sqrt{CM^2 + MQ^2} = \sqrt{x^2 + 12}$.

To compute FP, we note that since $AD \parallel QP \parallel BE$, we get that QP is the weighted average of AD and BE:

$$QP = \frac{AQ \cdot BE + BQ \cdot AD}{AB} = \frac{(2+x) \cdot 1 + (2-x) \cdot 2}{4} = \frac{6-x}{4}.$$

Therefore, we have that

$$\begin{aligned}\frac{FP}{CQ} &= 1 + \frac{CF + PQ}{CQ} \\ &= 1 + \frac{3 + \frac{6-x}{4}}{\sqrt{x^2+12}} = 1 + \frac{18 + x}{4\sqrt{x^2+12}}\end{aligned}$$

We then seek to maximize $\frac{x+18}{4\sqrt{x^2+12}}$. We show two ways to do so.

- **Method 1: Cauchy-Schwarz.** By Cauchy-Schwarz inequality, we have

  $$(x^2 + 12)(1 + 27) \geq (x + 18)^2,$$

  which implies that

  $$\frac{x+18}{4\sqrt{x^2+12}} \leq \frac{\sqrt{28}}{4} = \frac{\sqrt{7}}{2},$$

  and the equality holds when $\frac{x^2}{1} = \frac{12}{27}$, or $x = \frac{2}{3}$.

©2026 HMMT

- **Method 2: Calculus.** Let $f(x) = \frac{x+18}{4\sqrt{x^2+12}}$. We find the first derivative of f using quotient rule.

$$\begin{aligned} f'(x) &= \frac{1}{4} \cdot \frac{\sqrt{x^2+12} - (x+18) \cdot \frac{2x}{2\sqrt{x^2+12}}}{x^2+12} \\ &= \frac{1}{8} \cdot \frac{(x^2+12) - x(x+18)}{(x^2+12)^{3/2}} \\ &= \frac{1}{8} \cdot \frac{12-18x}{(x^2+12)^{3/2}}, \end{aligned}$$

  which is equal to 0 if and only if $x = \frac{2}{3}$. Furthermore, f is increasing in $\left(-\infty, \frac{2}{3}\right)$ and decreasing $\left(\frac{2}{3}, \infty\right)$, which implies that f has the only global maximum at $x = \frac{2}{3}$, and the maximum value is $f(x) = \frac{\sqrt{7}}{2}$.

Combining everything, the maximum value of $[DEF]$ is

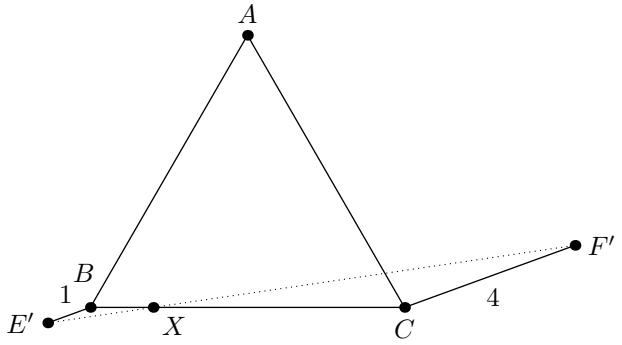
$$[ABC] \cdot \max \frac{DP}{AQ} = (4\sqrt{3}) \cdot \left(1 + \frac{\sqrt{7}}{2}\right) = \boxed{4\sqrt{3} + 2\sqrt{21}}.$$

Solution 3: Recall from Solution 1 and Solution 2 the following claim.

Claim 1. Lines $\overline{AD}$, $\overline{BE}$ and $\overline{CF}$ are parallel with rays $\overrightarrow{AD}$ and $\overrightarrow{BE}$ pointing in one direction, and $\overrightarrow{CF}$ pointing in the opposite direction.

Let E' be the point on ray $\overrightarrow{BE}$ with $BE' = 1$, and let F' be the point on ray $\overrightarrow{CF}$ with $CF' = 4$. Then, $\triangle AE'F'$ is a translation of $\triangle DEF$ by 1 unit in the direction $\overrightarrow{DA}$, so it suffices to maximize the area of $\triangle AE'F'$.

Let X be the point on segment $\overline{BC}$ such that $\frac{XB}{XC} = \frac{1}{4}$.



Claim 2. Segment $\overline{E'F'}$ always passes through the fixed point X; moreover, $\frac{XE'}{XF'} = \frac{1}{4}$.

Proof. Since $\frac{E'B}{F'C} = \frac{BX}{CX} = \frac{1}{4}$ and $\angle E'BX = \angle F'CX$ (from $\overline{BE'} \parallel \overline{CF'}$), by SAS similarity, we have that $\triangle E'BX \sim \triangle F'CX$. In particular, $\angle BXE' = \angle CXF'$, so E', X and F' are collinear. Moreover, we have $\frac{XE'}{XF'} = \frac{BX}{CX} = \frac{1}{4}$, as desired. □

The previous claim implies that $[AE'F'] = 5[AE'X]$. So, it suffices to maximize $[AE'X]$. Since $\overline{AX}$ is fixed, it suffices to maximize the distance from E' to $\overline{AX}$. We have

$$\operatorname{dist}(E', \overline{AX}) \leq \operatorname{dist}(E', B) + \operatorname{dist}(B, \overline{AX}) = 1 + \frac{2[ABX]}{AX},$$

©2026 HMMT

with equality when $\overline{E'B} \perp \overline{AX}$ and E' lies on the opposite side of $\overline{AB}$ as X. Therefore, we have

$$[DEF] = [AE'F'] = 5[AE'X] = \frac{5(AX)(\text{dist}(E', \overline{AX}))}{2} \leq 5[ABX] + \frac{5AX}{2}.$$

We compute $[ABX] = \frac{[ABC]}{5} = \frac{4\sqrt{3}}{5}$ and

$$AX = \sqrt{\left(\frac{6}{5}\right)^2 + \left(2\sqrt{3}\right)^2} = \frac{4\sqrt{21}}{5}.$$

Therefore, our final answer is

$$5[ABX] + \frac{5AX}{2} = \boxed{4\sqrt{3} + 2\sqrt{21}}.$$

©2026 HMMT